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Simulation of Decaying Two-Dimensional Turbulence Using Kinetically Reduced Local Navier-Stokes Equations

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Abstract: Kinetically Reduced Local Navier-Stokes (KRLNS) equations are applied for two-dimensional (2-D) simulation of doubly periodic shear layers and decaying homogeneous isotropic turbulence in order to demonstrate the capability to capture the correct transient behavior. The numerical results obtained by the KRLNS equations are compared with those obtained by the artificial compressibility method (ACM), the lattice Boltzmann method (LBM) and the pseudo-spectral method (PSM). The divergence as a function of time in the KRLNS method is compared with that of the ACM. It is confirmed that the KRLNS method can capture the correct transient behavior without use of sub-iterations due to a smoothing effect.

Keywords: Unsteady Incompressible Viscous Flows, Kinetically Reduced Local Navier-Stokes Equations, Artificial Compressibility Method, lattice Boltzmann Method, Pseudo-Spectral Method.

1 Introduction

For unsteady incompressible viscous flows, it is essential to solve the Poisson equation at each time step, which is quite time consuming. Development of more efficient approach is still needed before large-scale computation of complicated fluid dynamic problems.

Lattice Boltzmann Method (LBM) [1] is a good candidate and has been used extensively. Recently, it was proposed to use Artificial Compressibility Method (ACM) [2], without sub-iteration for unsteady flow computations, in addition, an alternative thermodynamic description of incompressible fluid flows was suggested in the form of Kinetically Reduced Local Navier-Stokes (KRLNS) equations [3], which can capture the correct time dynamics without sub-iterations. It is necessary to investigate the capability to capture the correct transient behavior of these approaches, LBM, ACM, and the KRLNS equations.

In this paper, in order to investigate the capability to capture the correct transient behavior of the KRLNS equations, two-dimensional numerical simulations of doubly periodic shear layers and decaying homogeneous isotropic turbulence are carried out and the results are compared with the solutions obtained by ACM, LBM and Pseudo-Spectral Method (PSM) [4].

2 Kinetically Reduced Local Navier-Stokes Equations

The classical incompressible Navier-Stokes equations consist of the equation for the momentum

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$
$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{\partial p}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (1)$$

and the continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (2)$$

where t is the physical time, $\mathbf{u}$ is the fluid velocity, p is the pressure and Re is the Reynolds number.
In the ACM, the pseudo-time derivatives of the velocity and pressure are introduced into Eq. (1) and (2), respectively for coupling between the pressure and the velocity. The incompressible Navier-Stokes equations and the continuity equation for the ACM are then written as

$$\frac{\partial u}{\partial \tau}+\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$
$$\frac{\partial v}{\partial \tau}+\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{\partial p}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (3)$$

$$\frac{1}{\beta}\frac{\partial p}{\partial \tau}=-\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right), \qquad (4)$$

where τ is the pseudo-time and β is the artificial compressibility parameter. To satisfy the continuity equation (2), the sub-iterations at each time step is mandatory.
In the form of KRLNS equations, the pressure equation (4) is replaced by

$$\frac{\partial G}{\partial t}=-\frac{1}{Ma^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 G}{\partial y^2}\right), \quad p=G+\frac{u^2+v^2}{2} \qquad (5)$$

,where Ma is the Mach number, G is the grand potential. Retaining the term $(\partial^2 G/\partial x^2+\partial^2 G/\partial y^2)/\mathrm{Re}$ is crucial for capturing the correct transient behavior without sub-iterations.

3 Numerical Method

In the numerical method for solving the KRLNS equations, central difference scheme is used for the spatial discretization in both advection and diffusion terms, and 4 stage Runge-Kutta method is used for the time integration. For the ACM with sub-iteration, FDS method is used in the advection term, and central difference scheme is used in the diffusion term. Backward Euler method is used in the physical time, and forward Euler method is used in the pseudo-time integration.

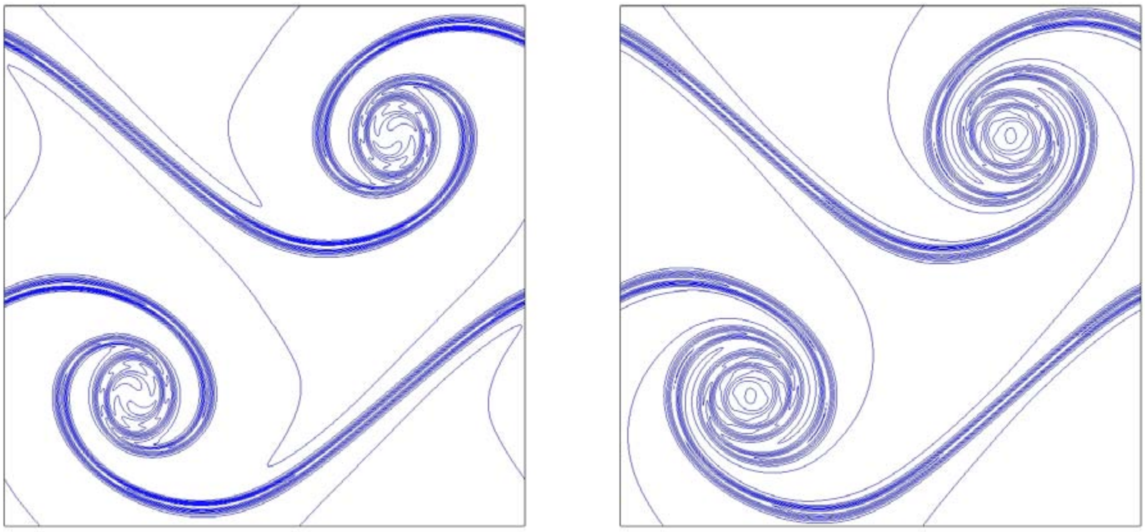
4 Doubly Periodic Shear Layers

The flow simulation of the evolution of double shear layers governed by the Navier-Stokes equations is implemented on a unit square [0,1] × [1,0] with periodic boundary condition. The initial conditions are given by

$$u = \begin{cases} \tanh(\kappa(y-0.25)), & y \le 0.5 \\ \tanh(\kappa(0.75-y)), & y > 0.5 \end{cases} \quad (6)$$
$$v = \delta \sin(2\pi(x+0.25))$$

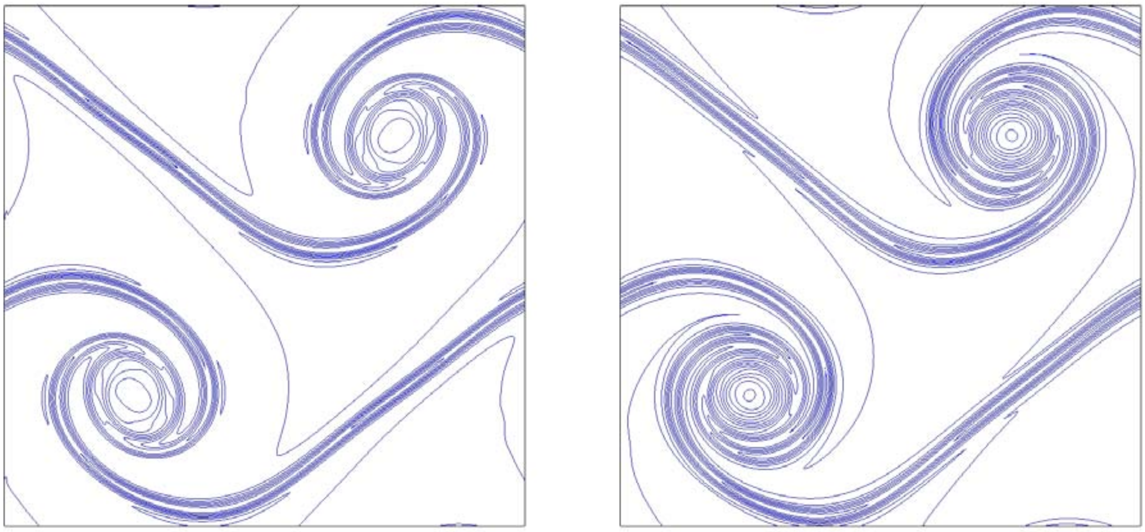
,where κ is the shear layer width parameter, and δ is the magnitude of the initial perturbation. Here, $\kappa = 80$, and $\delta = 0.05$ are used. This case is the shear layers roll up into large vortical structures as the flow evolves.

The vorticity contour plot obtained by the KRLNS equations at Re = 10000 on a uniform 512×512 Cartesian grid is shown in Figure 1, while Figure 2 shows that obtained by the ACM. The adjustable parameters are Ma = 0.2, 0.02, $\Delta t = 1 \times 10^{-5}$ for the KRLNS equations and β=1, 10, $\Delta t = 1 \times 10^{-4}$ for the ACM. Regarding the number of sub-iterations for the ACM, 30 times at each time step is imposed. The result obtained by the LBM is also shown in Figure 3, which is used as the reference solution. The contour plots of the KRLNS equations and the ACM compared to that of the LBM are shown in Figure 4 and Figure 5, respectively. The good agreement among the three simulations of the KRLNS equations (Ma = 0.02), the ACM (β=1, 10) and the LBM is obtained. The divergence as a function of time for the KRLNS equations is shown in Figure 6, while Figure 7 shows that obtained by the ACM. It is confirmed that the KRLNS can keep the divergence fluctuation at the 10^{-3} level, while that in the ACM is at the 10^{-2} level.



(a) Ma=0.2　　　　(b) Ma=0.02

Figure 1: Vorticity contours obtained by KRLNS at time t=1.



(a) β =1 (b) β =10

Figure 2: Vorticity contours obtained by ACM at time t=1.

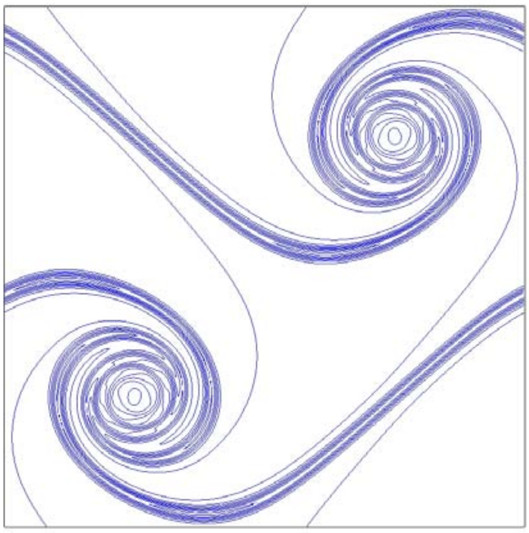
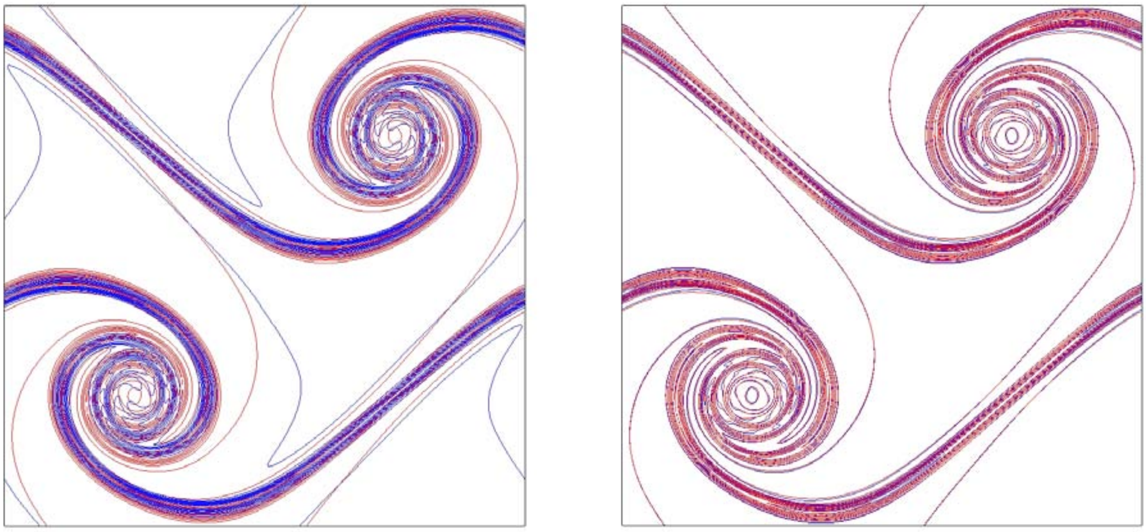
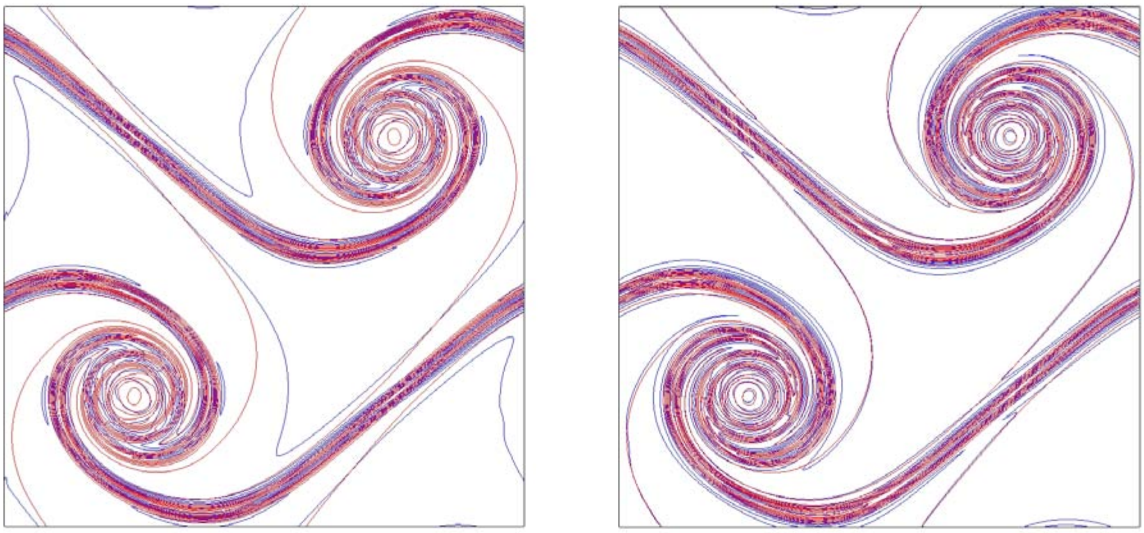


Figure 3: Vorticity contours obtained by LBM at time t=1.



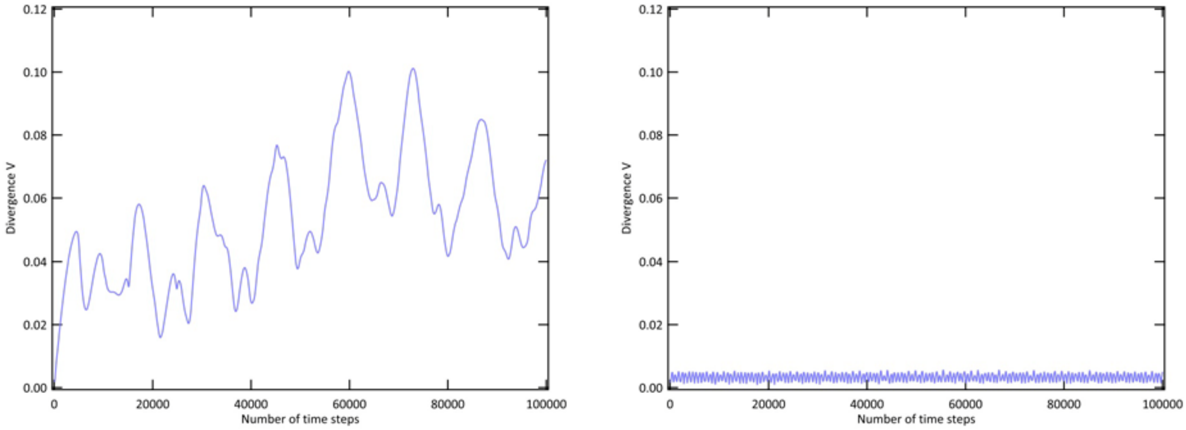
(a) Ma=0.2 (b) Ma=0.02

Figure 4: Comparison of vorticity contours between KRLNS (blue) and LBM (red) at time t=1.



(a) β =1 (b) β =10

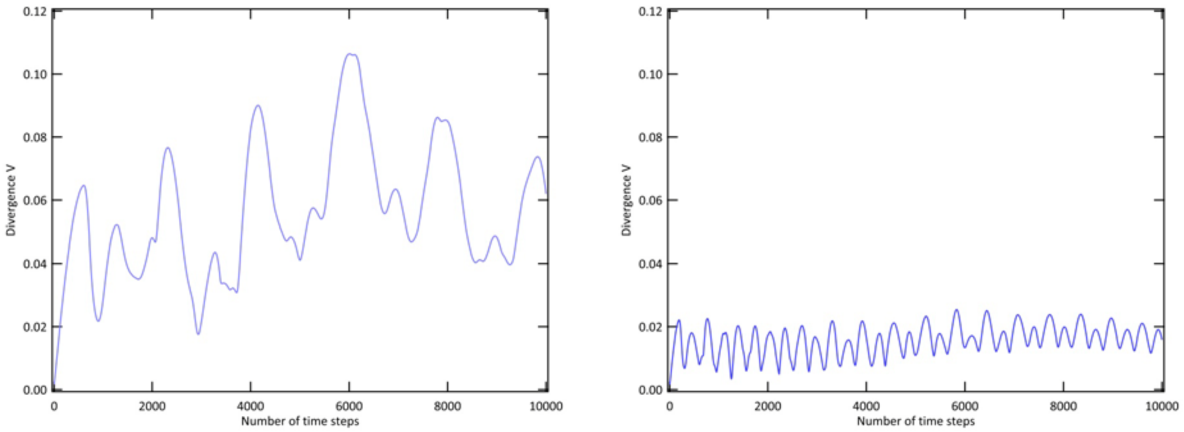
Figure 5: Comparison of vorticity contours between ACM (blue) and LBM (red) at time t=1.



(a) Ma=0.2

(b) Ma=0.02

Figure 6: Divergence histories obtained by KRLNS.



(a) $\beta=1$

(b) $\beta=10$

Figure 7: Divergence histories obtained by ACM.

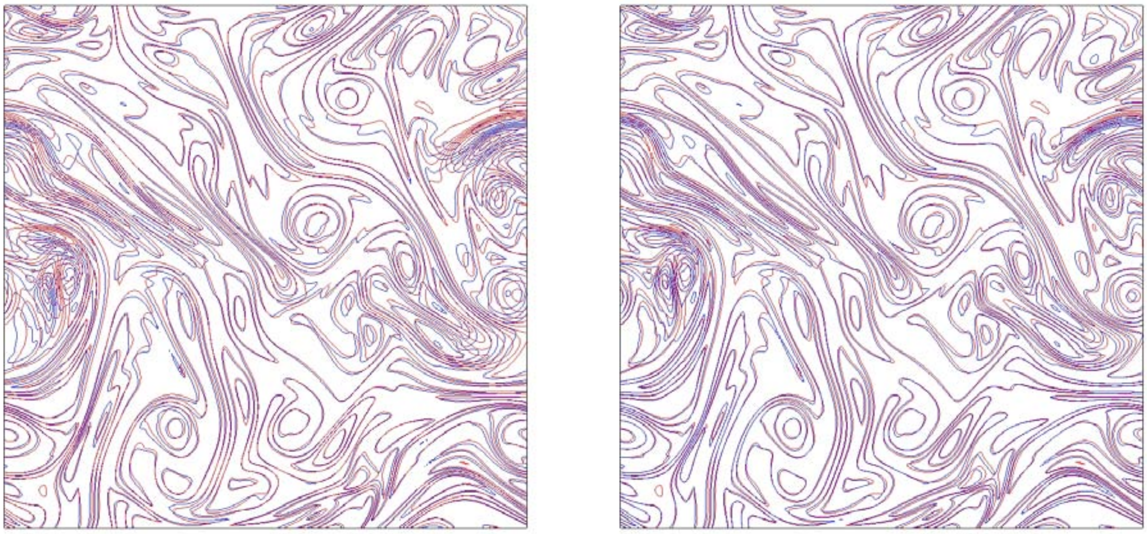
5 Homogeneous Isotropic Turbulence

The flow simulation of homogeneous isotropic turbulence is implemented on a unit square [0,1] × [1,0] with periodic boundary condition. The initial condition of the velocity is randomly determined by satisfying the relation,

$$E(k)=\frac{1}{2}\sum_{|k'-k|\le 1/2}\frac{|\tilde{\omega}(k_1,k_2)|^2}{k'^2}=\frac{2}{3}k\exp\left(-\frac{2}{3}k\right) \qquad (7)$$

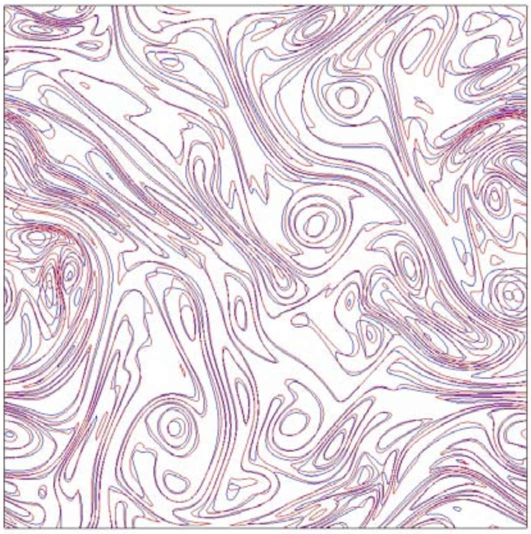
where $\tilde{\omega}$ denotes the vorticity in the Fourier space, $k'^2=k_1^2+k_2^2$, and k_1 and k_2 are the wave numbers.

The vorticity contour plot obtained by the KRLNS equations at Re = 10000 on a uniform 256×256 Cartesian grid is shown in Figure 8, while Figure 9 shows that obtained by the ACM. The adjustable parameters are Ma = 0.2, 0.02, $\Delta t = 1 \times 10^{-4}$ for the KRLNS equations and β=1, 10, $\Delta t = 1 \times 10^{-4}$ for the ACM. Regarding the number of sub-iterations for the ACM, 30 times at each time step is imposed. The result obtained by the LBM is also shown in Figure 10. These results are compared to the reference solution obtained by the PSM, because it is a standard approach to the direct simulation of turbulence. The good agreement among the four simulations of the KRLNS equations (Ma = 0.02), the ACM (β=1, 10), the LBM and the PSM is obtained. The divergence as a function of time for the KRLNS equations and the ACM is shown in Figure 11 and Figure 12, respectively. It is confirmed that the KRLNS approach can keep the divergence fluctuation at the 10^{-2} level, while that in the ACM is much larger. The computational time of the KRLNS equations is about 13 times faster than that of the ACM for the same time interval.

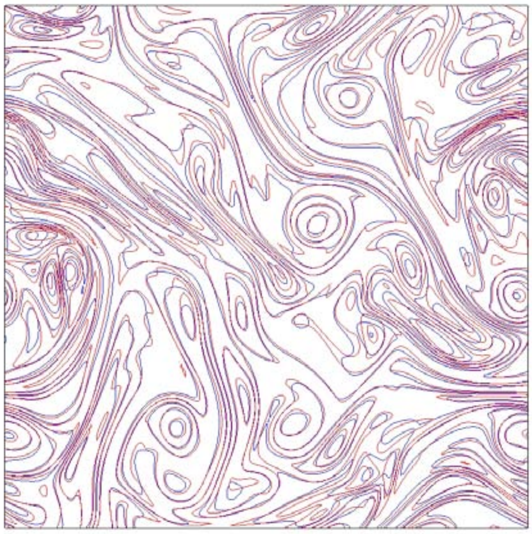


(a) Ma=0.2 (b) Ma=0.02

Figure 8: Comparison of vorticity contours between KRLNS (blue) and PSM (red) at time t=1.



(a) β =1



(b) β =10

Figure 9: Comparison of vorticity contours between ACM (blue) and PSM (red) at time t=1.

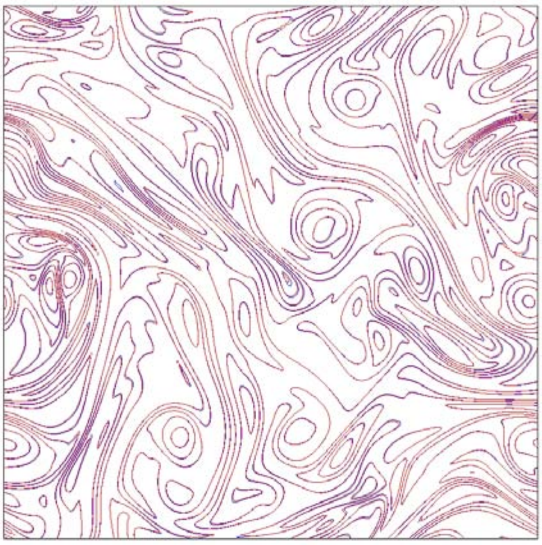
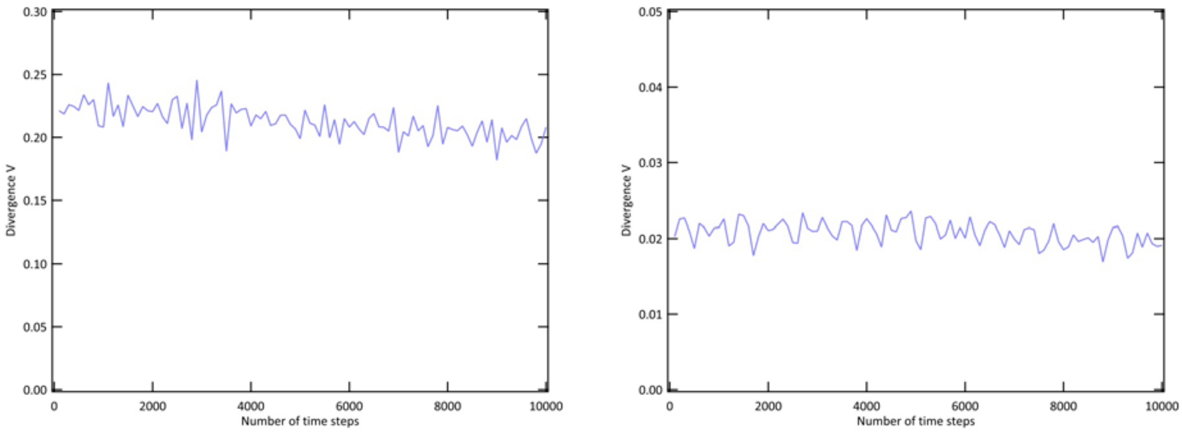
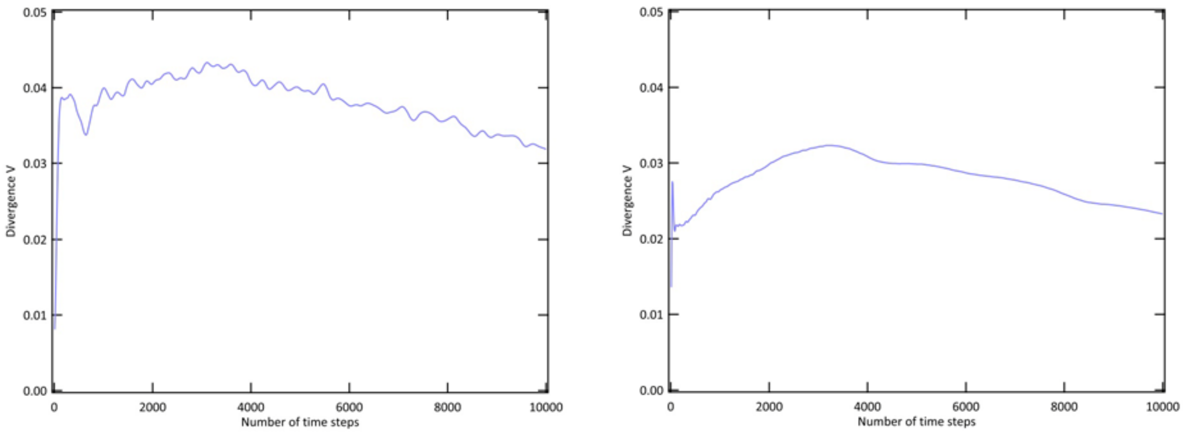


Figure 10: Comparison of vorticity contours between LBM (blue) and PSM (red) at time t=1.



(a) Ma=0.2 (b) Ma=0.02

Figure 11: Divergence histories obtained by KRLNS.



(a) β =1 (b) β =10

Figure 12: Divergence histories obtained by ACM.

6 Conclusions

Kinetically Reduced Local Navier-Stokes (KRLNS) equations are applied for simulations of two test cases in order to demonstrate the capability to capture the correct transient behavior. It is found that the KRLNS approach can capture the correct transient behavior without sub-iterations. The solution obtained by the KRLNS equations was in excellent agreement with those of the other standard approaches. The KRLNS approach can keep the divergence fluctuation at smaller level by giving small Mach number than that of the ACM with sub-iterations. The computational time of the KRLNS equations is faster than that of the ACM.

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