Seventh International Conference on
Computational Fluid Dynamics (ICCFD7),
Big Island, Hawaii, July 9-13, 2012

ICCFD7-4205

An Implicit Hermite WENO Reconstruction-Based Discontinuous Galerkin Method on Tetrahedral Grids

Yidong Xia[*] and Robert Nourgaliev[**]
Corresponding author: yxia2@ncsu.edu

* North Carolina State University, Raleigh, NC 27695, USA
** Idaho National Laboratory, Idaho Falls, ID 83415 USA.

Abstract: An Implicit Reconstructed Discontinuous Galerkin method, IRDG(P_1P_2), is presented for solving the compressible Euler equations on tetrahedral grids. In this method, a quadratic polynomial (P_2) solution is first reconstructed using a least-squares method from the underlying linear polynomial (P_1) DG solution. By taking advantage of the derivatives in the DG formulation, the stencils used in the reconstruction involve only von Neumann neighborhood (adjacent face-neighboring cells) and thus are compact and consistent with the underlying DG method. The final P_2 solution is then obtained using a WENO reconstruction, which is necessary to ensure stability of the RDG(P_1P_2) method. A matrix-free GMRES (generalized minimum residual) algorithm is presented to solve the approximate system of linear equations arising from Newton linearization. The LU-SGS (lower-upper symmetric Gauss-Seidel) preconditioner is applied with both the simplified and approximate Jacobian matrices. The numerical experiments on a variety of flow problems demonstrate that the developed IRDG(P_1P_2) method is able to obtain a speedup of at least two orders of magnitude than its explicit counterpart, maintain the linear stability, and achieve the designed third order of accuracy: one order of accuracy higher than the underlying second-order DG(P_1) method without significant increase in computing costs and storage requirements. It is also found that a well approximated Jacobian matrix is essential for the IRDG method to achieve fast converging speed and maintain robustness on large-scale problems.

Keywords: Discontinuous Galerkin, Reconstruction Method, WENO, Compressible Flows, Implicit Method, Tetrahedral Grids.