Multi-dimensional Limiting Strategy for Arbitrary Higher-order Discontinuous Galerkin Methods in Inviscid and Viscous Flows

J. S. Park* and C. Kim*
Corresponding author: chongam@snu.ac.kr

* Seoul National University, Korea.

Abstract: The present paper deals with the multi-dimensional limiting process (MLP) for arbitrary higher-order discontinuous Galerkin (DG) methods to compute compressible inviscid and viscous flows. MLP, which has been quite successful in finite volume methods (FVM), is extended into DG methods for hyperbolic conservation laws. We firstly formulate the robust and accurate hierarchical MLP limiting strategy for DG-$Pn(n \geq 2)$ reconstruction. Secondly, the resulting method, called the DG-MLP method, is applied to various inviscid and viscous flows. In the final paper, detailed numerical procedure for the hierarchical DG-MLP formulation and extensive numerical test cases to demonstrate its numerical accuracy and robustness will be given.

Keywords: Higher-order Methods, Multi-dimensional Limiting Process, Arbitrary Higher-order DG Methods, Inviscid and Viscous Flows.

1 Introduction

Multi-dimensional limiting process (MLP) has been developed quite successfully in FVM. Compared with traditional limiting strategies, such as TVD or ENO-type schemes, MLP effectively controls unwanted oscillations particularly in multi-dimensional flow situations. The previous work [1] clearly shows that the MLP method possesses superior characteristics in terms of accuracy, robustness and efficiency in inviscid and viscous computations within FVM framework.

Recently, MLP concept has been successfully extended into DG methods for compressible flows (or hyperbolic conservation laws) on unstructured grids. This approach, called the DG-MLP methods, are able to accurately capture complex compressible multi-dimensional flow structure without yielding unwanted oscillations [2]. Particularly, it can provide multi-dimensional monotonic solutions without compromising formal order of accuracy in smooth region. Based on the previous progresses, a robust and accurate but simple form of DG-MLP limiting is provided so that it can handle complex vortical structures in viscous flows very accurately, even if such flow features interact with strong shocks.

2 Hierarchical MLP Limiting for DG-Pn Reconstruction

The proposed limiting strategy starts from the MLP condition, which requires to control the distribution of both cell-centered and cell-vertex physical properties to mimic a multi-dimensional nature of flow physics. For a linear reconstruction, the MLP-u slope limiters satisfy the maximum principle on the MLP stencil to ensure multi-dimensional monotonicity while maintaining the second order of accuracy on unstructured meshes [1].

The MLP strategy can be readily extended into higher-order methods, such as discontinuous Galerkin method. To maintain formal accuracy across smooth extrema without creating spurious oscillations across discontinuous region, limiting should be activated only on the troubled-cells. In FVM with $P1$ reconstruction, the MLP condition is enough to identify and control the maximum-principle-violating cells. For higher-order $Pn(n \geq 2)$ reconstruction, the MLP condition is not sufficient, and thus an extended MLP concept, call the augmented MLP (A-MLP) condition, is introduced to handle higher-order smooth distribution. On top of it, local smooth extrema at vertex point, obtained by DG-Pn reconstruction, can be efficiently monitored by decomposing it into linear part and higher-order part. By combining the A-MLP condition and the behavior of local smooth extrema, the hierarchical MLP limiting for DG-Pn reconstruction can be obtained. The DG-MLP limiting procedure is then successively applied from Pn to $P(n-1)$ mode, and finally the MLP slope limiter for P1 mode.

The following table presents the grid refinement test for isentropic vortex advection problem at $t = 10.0$, and it clearly confirms the desired grid-convergence characteristic of the DG-MLP methods. Figure 1 compares the density contours for the forward facing step problem, as a standard benchmark test for higher-order schemes.

	$1/h$	L^∞	Order	L^1	Order
MLP-u1	20	4.8255E-02		2.9957E-03	
DG-P1	40	1.0472E-02	2.20	6.4309E-04	2.22
	80	1.9421E-03	2.43	1.1591E-04	2.47
	160	3.9996E-04	2.28	2.2460E-05	2.37
MLP-u1	20	3.3952E-03		2.9147E-04	
DG-P2	40	3.2963E-04	3.36	2.0734E-05	3.81
	80	3.6873E-05	3.16	1.6946E-06	3.61
	160	4.3960E-06	3.07	1.6527E-07	3.36
MLP-u1	20	3.3145E-04		2.0910E-05	
DG-P3	40	1.7153E-05	4.27	1.0618E-06	4.30
	80	1.0891E-06	3.98	5.6998E-08	4.22
	160	6.1056E-08	4.16	3.5888E-09	3.99

Table 1: Grid refinement test for the evolution of an isentropic vortex at $t = 10.0$.

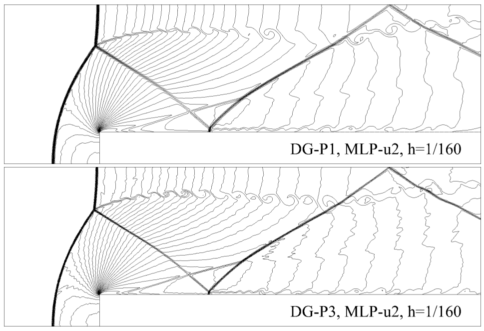

Figure 1: Comparison of density contours for the Mach 3 wind tunnel with a step.

3 Conclusions

The DG-MLP methods are able to capture complex multi-dimensional flow structures without yielding unwanted oscillations for compressible flow. By employing a way to handle the diffusion term, such as LDG, the proposed approach is going to apply compressible viscous flows. Detailed procedure to derive the hierarchical DG-MLP formulation for compressible viscous flows and extensive numerical test cases will be given in the full paper.

References

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