A Reconstructed Discontinuous Galerkin Method Based on a Hierarchical Hermite WENO Reconstruction for Compressible Flows on Tetrahedral Grids

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Abstract: A hierarchical Hermite WENO reconstruction-based discontinuous Galerkin method, designed not only to enhance the accuracy of discontinuous Galerkin method but also to avoid spurious oscillation in the vicinity of discontinuities, is developed for compressible flows on tetrahedral grids. In this method, a quadratic polynomial solution is first reconstructed from the underlying linear polynomial discontinuous Galerkin solution using a least-squares method. A Hermite WENO reconstruction is then performed to obtain the final representation of the quadratic polynomial solution. The developed RDG method is used to compute a variety of flow problems on tetrahedral meshes. The numerical experiments demonstrate that this Hermite WENO reconstruction-based RDG(P1P2) method is able to achieve the designed third-order accuracy, and outperforms the third-order DG method DG(P2) in terms of both computing costs and storage requirements

Keywords: Discontinuous Galerkin Methods, WENO Reconstruction, Compressible Flows.

1 Introduction and Numerical Method

The discontinuous Galerkin (DG) methods have recently become popular for the solution of systems of conservation laws, and are widely used in computational fluid dynamics, computational acoustics, computational electromagnetics, and computational magneto-hydrodynamics due to many attractive features they possess. However, the DG methods have a number of weaknesses that need to be addressed before they can be routinely used for flow problems of practical interest in a production environment. In particular, how to effectively eliminate spurious oscillations in the presence of strong discontinuities, and how to reduce the computing costs for the DG methods remain the two most challenging and unresolved issues in the DGM. Indeed, compared to the finite element methods and finite volume methods, the DG methods require solutions of systems of equations with more unknowns for the same grids. Consequently, these methods have been recognized as expensive in terms of both computational costs and storage requirements. In order to reduce high costs associated with the DG methods, a new family of reconstructed DG methods, termed RDG(PnPm), has been developed in the literature[1-2], where Pn indicates that a piecewise polynomial of degree of n is used to represent a DG solution, and Pm represents a reconstructed polynomial solution of degree of m ($m \geq n$) that is used to compute fluxes. The RDG(PnPm) schemes are designed to enhance the accuracy of the discontinuous Galerkin method by increasing the order of the underlying polynomial solution. The beauty of RDG(PnPm)schemes is that they provide a unified formulation for both finite volume and DG methods, and contain both classical finite volume and standard DG methods as two special cases of RDG(PnPm) schemes, and thus allow for a direct efficiency comparison. When $n=0$, i.e. a piecewise constant polynomial is used to represent a numerical solution, RDG(P0Pm) is nothing but classical high order finite volume schemes, where a polynomial solution of degree m ($m \geq 1$) is reconstructed from a piecewise constant solution. When $m=n$, the reconstruction reduces to the identity operator, and RDG(PnPn) scheme yields a standard DG(Pn) method. The objective of the effort discussed in this paper is to develop a RDG(P1P2) method based on a Hermite WENO reconstruction for solving compressible flows on unstructured grids. This HWENO-based RDG method is designed not only to reduce the high computing costs for the DG methods, but also to avoid spurious oscillations in the vicinity of strong discontinuities, thus effectively overcoming the two shortcomings of the DG methods. In this RDG(P1P2) method, the reconstruction of a quadratic solution consists of two steps: (1) the second derivatives on each cell are first reconstructed from the solution variables and their first derivatives using the solution variables and their first derivatives from adjacent face-neighboring cells via a strong interpolation; (2) the final second derivatives on each cell are then obtained using a WENO strategy based on the reconstructed second derivatives on the cell itself and its adjacent face-neighboring cells. This reconstruction scheme, by taking advantage of handily available and yet valuable information namely the gradients in the context of the DG methods, only involves the von Neumann neighborhood and thus is compact, simple, robust, and flexible. Since the second derivatives are based on the WENO reconstruction, the resulting RDG method can achieve non-linear stability with the aid of a WENO reconstruction for the first derivatives using a hierarchical reconstruction[3], which will be reported in the full manuscript. As the underlying DG method is second-order, and the basis functions are at most linear functions, fewer quadrature points are then required for both domain and face integrals, and the number of unknowns (the number of degrees of freedom) remains the same as for the DG(P1). Consequently, this RDG method is more efficient than the third order RDG(P2P2) method. The developed RDG method is used to compute a variety of flow problems on unstructured tetrahedral grids to demonstrate its accuracy, efficiency, and robustness. The numerical results indicate that this RDG method is able to obtain a third-order accurate solution at a slightly higher cost than its second-order DG method and provide a significant increase in performance over the third order DG method in terms of computing costs and storage requirements. As two illustrative examples, a subsonic flow past a sphere is chosen to demonstrate a formal order of the convergence rate of the RDG(P1P2) method and its superior performance over the RDG(P0P1) and RDG(P1P1) methods (Figure 1) and a transonic flow past a wing-pylon-nacelle configuration is presented to demonstrate that this RDG method can effectively eliminate spurious oscillations in the vicinity of strong discontinuities. (Figure 2)

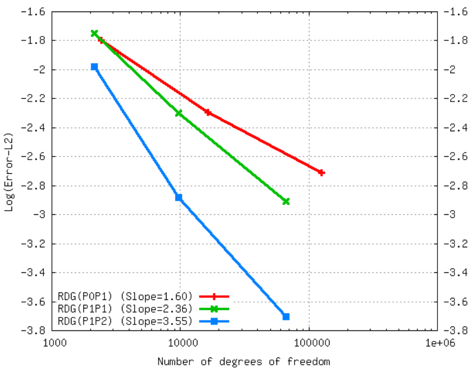

Figure 1: Convergence history of RDG(P0P1), RDG(P1P1) and RDG(P1P2) solutions for subsonic flow past a sphere.

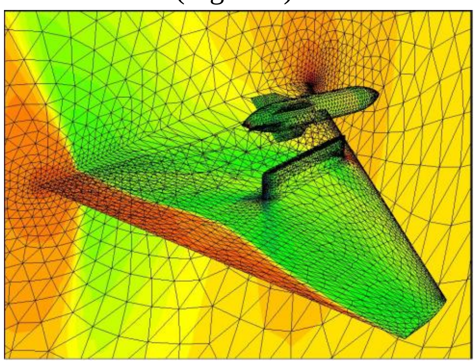

Figure 2: Computed Pressure Contours for a transonic flow past a wing/pylon/nacelle configuration.

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