# High-Fidelity Constrained Optimization of a Pitching and Plunging Airfoil

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**Abstract:** We present the results from a series of optimizations of the pitching and plunging motion of a NACA0012 airfoil. The optimization objective is to minimize the aerodynamic input power as a function of the angle of attack and the parameters of the pitching and plunging motion. Constraints are imposed to fix the values of time-averaged lift and thrust coefficients. The lift, thrust and aerodynamic power are computed with an unsteady Navier-Stokes solver, enabling high-fidelity solutions that capture the complex, unsteady flow phenomenon. Optimizations are carried out using a gradient-based optimization algorithm. Results are presented and discussed for a range of thrust and lift constraint values.

*Keywords:* Computational Fluid Dynamics, Optimization, Unsteady Flow, Flapping Wings.

## 1 Introduction

There has been a significant increase in research interest in the aerodynamics of flapping wings within the last decade as a result of both increasing interest in small-scale aircraft such as microaerial vehicles (MAVs) and because computing capabilities are approaching the necessary levels of performance simulate the unsteady, viscous flow physics associated with flapping wings.

A common condition for a flying vehicle is the steady, level flight case where there is an equilibrium between thrust and drag and between lift and weight. For a fixed-wing craft this condition is achieved by through the proper combination of engine thrust and vehicle angle-of-attack. This condition is more complex for a flapping wing vehicle, however, since lift and thrust are both produced through the motion of the wing. Energy efficiency is also especially important for small flying vehicles due to limited battery energy density. Therefore, an important objective is to minimize the power expended during steady, level flight.

We attempt to provide some insight into the flapping motions and flow physics that lead efficient steady flight by performing constrained optimizations to minimize power for a range of prescribed thrust and lift values.

## 2 Problem Statement

Constrained power minimizations are performed by coupling a high-fidelity unsteady Navier-Stokes solver with a gradient-based optimization algorithm. Details of the solver, the optimization algorithm and the formulation of the optimization problem are given in the follow sections.

#### Flow Solver

The flow solvers are based on the low-dissipation kinetic energy preserving (KEP) finite volume scheme. The kinetic energy preserving property of this scheme allows stability to be maintained with little or no artificial dissipation. This property is especially desirable for vortex dominated flows such as flapping flight since artificial dissipation tends to quickly and unnaturally dampen complex flow features. This code has been specifically developed to simulate an oscillating airfoil and to be integrated into an optimization framework. The full details of this code can be found in the work of Allaneau *et. al.*[1]. In all cases we consider a pitching and plunging airfoil with Re = 1,850 and M = 0.2.

#### **Optimization Problem**

We consider a constrained, non-linear optimization problem of the form

$$\begin{array}{ll} \text{minimize} & P_{mech}(\mathbf{x}) \\ \text{subject to} & C_L = C_{L_{target}} \\ C_T = C_{T_{target}} \end{array} \tag{1}$$

where  $P_{mech}$  is the mechanical power required to pitch and plunge the airfoil, **x** contains the variables for angle of attack, frequency, pitch and plunge amplitude and phase difference, and  $C_L$  and  $C_T$  are the time-averaged lift and thrust coefficients. In all cases the average lift and thrust are computed by integrating over a single flapping cycle after a suitable number of periods have elapsed to allow the flow to reach a quasi-periodic state.

Optimizations are carried out using the SNOPT software package, which is based on the gradient-based sequential quadratic programming (SQP) algorithm [2]. Gradients are evaluated using a finite difference approximation.

## 3 Conclusion and Future Work

Optimizations are on-going at this time. Full results and conclusions will be discussed in the final paper.

### References

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