Dynamic Model of Vortex Cavitation based on Axisymmetric Navier-Stokes Equation

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Abstract: In this study, a new dynamic model of vortex cavitation is proposed as an approximate solution of the unsteady axisymmetric Navier-Stokes equation. This model provides unsteady behaviors of a sub-surface vortex, i.e., development of the vortex, occurrence of the vortex cavitation, evolution and contraction of the cavitating radius. In addition, the effect of surface tension is considered in the model. The simulation results under steady and fluctuating far-field pressure conditions show that the model has the ability to simulate the vortex cavitation.

Keywords: Vortex Cavitation, Axisymmetric Navier-Stokes Equation, Cavitation Model, Surface Tension, Cavitating Radius.

1 Introduction

The vortex cavitation due to a sub-surface vortex can be observed in a lot of industrial scenes, such as a pump intake or a tip of impeller blade. A great deal of studies have been conducted to suppress the cavitation occurrence because noise, vibration and/or erosion can be induced by the vortex cavitaion. Also in our study on sodium-cooled fast reactors, the vortex cavitaion near the intake mouth of the outlet pipe from the reactor core is investigated experimentally and numerically to establish the cavitation-free design. However, the direct numerical simulation of the vortex cavitation has a great difficulty, that is, the cavitating radius is often very small to require an unrealistically fine mesh for an accurate numerical simulation. In such a case, it is preferable to simulate the vortex cavitation with a cavitation model. In past researches, the Rayleigh-Plesset (R-P) type cavitation model [1] is employed and the vortex cavitaion is simulated as a series of gas bubbles along the sub-surface vortex core. This type of numerical simulations gives some useful knowledge, however, physically more realistic model should be employed to simulate the vortex cavitation with high accuracy. Therefore, in this study, the authors propose a new modeling of the vortex cavitation based on the axisymmetric Navier-Stokes (N-S) equation. In concrete term, an approximate solution of the N-S equation is derived and the radial pressure distribution is calculated from the solution. Then, the cavitating radius is determined by comparison of the calculated pressure with vapor pressure. The surface tension is also considered in the model to calculate the cavitating radius accurately. Several simulations are performed under various far-filed and surface tension conditions to confirm the physical adequacy of the proposed cavitation model.

2 Dynamic Vortex Cavitaion Model

With the assumptions of the axisymmetric vortical flow along a sub-surface vortex core and the uniform axial velocity in radial direction, the new vortex cavitation model is obtained as an appropriate solution of the unsteady axisymmetric N-S equation. Namely, the circumferential velocity, v is written as

$$v = \frac{1}{1+T'\exp(-\alpha t)} \frac{\Gamma_\infty}{2\pi r} \left\{ 1 - \exp\left(-\frac{1}{1+T'\exp(-\alpha t)} \frac{r^2 - R^2}{r_0^2} \right) \right\}$$

where Γ_∞ is the circulation at infinity, T is the constant determined by an initial condition, α is the axial velocity gradient and r_0 is the specific radius of the vortex. R is the cavitating radius which is determined from the radial pressure distribution. In other words, the region where the pressure is smaller than the vapor pressure is identified as the cavitating region. The pressure distribution is calculated by the balance equation between centrifugal force, pressure gradient and surface tension. This dynamic cavitation model provides unsteady behaviors of the vortex cavitation, i.e., development of the sub-surface vortex, occurrence of the cavitation at the vortex core, evolution and contraction of the cavitating radius. In fact, when the simulation is performed from an initial very weak vortex with steady far-field pressure, the vortex develops to be strong enough to make the pressure at the vortex core lower than the vapor pressure and then, the cavitating radius grows rapidly with further development of the vortex until a terminal steady state. As for the effect of surface tension, the velocity distribution is almost the same regardless of the surface tension strength, however, the cavitating radius becomes smaller with the increase in the surface tension coefficient. Lastly, the simulation is performed with fluctuating far-field pressure. As shown in Figure 1, after the development stage of the vortex, the cavitating radius changes in synchronization with the far-field pressure fluctuation. From these simulation results, it is shown that the physics of the vortex cavitation is represented appropriately by the proposed cavitation model. Therefore, it can be concluded that the model has an adequate ability to reproduce the vortex cavitation due to a sub-surface vortex.

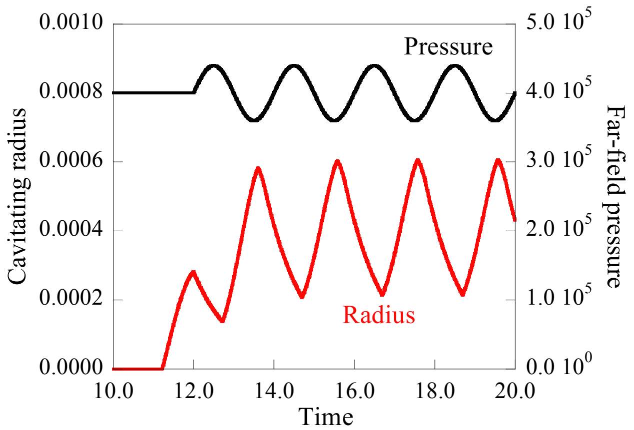

Figure 1: Time fluctuation of cavitating radius and far-field pressure.

3 Conclusion and Future Work

The several simulation results show that the proposed dynamic cavitation model provides physically appropriate behaviors of the vortex cavitation. This fact implies that the model is developed in a proper manner from the N-S equation. As a future work, the model will be incorporated into our three-dimensional CFD code which simulates gas-liquid two-phase flows by a high-precision volume-of-fluid algorithm. Then, the numerical simulations of sub-surface vortices are performed to validate the reproducibility of the vortex cavitation by the CFD code.

References

[1] M. S. Plesset and A. Prosperetti, Bubble dynamics and cavitation, Annual Review of Fluid Mechanics, 9:145-185, 1977.