Numerical simulations of two-dimensional turbulent thermal convection on the surface of a soap bubble

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Abstract: We present in this paper a stereographic projection based method to implement cartesian computations in order to simulate the behavior of a half bubble of soap.

Keywords: Two-dimensional turbulence, hemispheric simulations.

1 Introduction

The problem consists in performing the numerical simulations of a half bubble of soap located on a heated plate [1]. The gradient of temperature between the base and the top of the bubble generates plumes at the base that move up to the top. These plumes give rise to eddies that survive for several minutes eventually creating a two-dimensional turbulent thermal convective phenomenon. Our method consists in defining an appropriate stereographic projection in order to use classic numerical scheme defined for planar two-dimensional Navier-Stokes equations.

2 Mathematical model and numerical approximations

The equations for the two-dimensional thermal convection under the Boussinesq approximation can be written as:

$$\begin{cases} \dfrac{D\mathbf{u}}{Dt} = -\dfrac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} - \beta T\mathbf{g} \\ \nabla.\mathbf{u} = 0 \\ \dfrac{DT}{Dt} = cT + \kappa\Delta T \end{cases} \tag{1}$$

where $\dfrac{D*}{Dt} \equiv \dfrac{\partial *}{\partial t} + (\mathbf{u}.\nabla)*$ is the total time derivative, $\mathbf{u} = (u, v)$ denotes the velocity, p the pressure, ρ the mass density, ν the kinematic viscosity, β the coefficient of thermal expansion, $\mathbf{g}$ the gravity field, T the temperature of the fluid, c the parameter for the heat source and κ the coefficient of heat conductivity.

The stereographic projection is a particular mapping that allows us to project a sphere onto a plane. This mapping is bijective and preserves the angles: a cartesian grid on the plane corresponds to a structured grid on the surface of the sphere. The equations are solved in primitive variables with a multigrid procedure on the cartesian mesh. A penalization method is used to define the circular domain and a no-slip boundary condition is set on the equator, as well as a Dirichlet boundary condition for T. The discretization is done by a global second order scheme in time and space including an accurate third order approximation of convection terms.

3 Results and analysis

Any function with spherical symmetry can be easily written as a linear combination of special functions $\{Y_l^m\}$, the so-called spherical harmonics, $f(\theta,\phi) = \sum_{l=0}^{+\infty} \sum_{m=-l}^{+l} f_l^m Y_m^l(\theta,\phi)$.

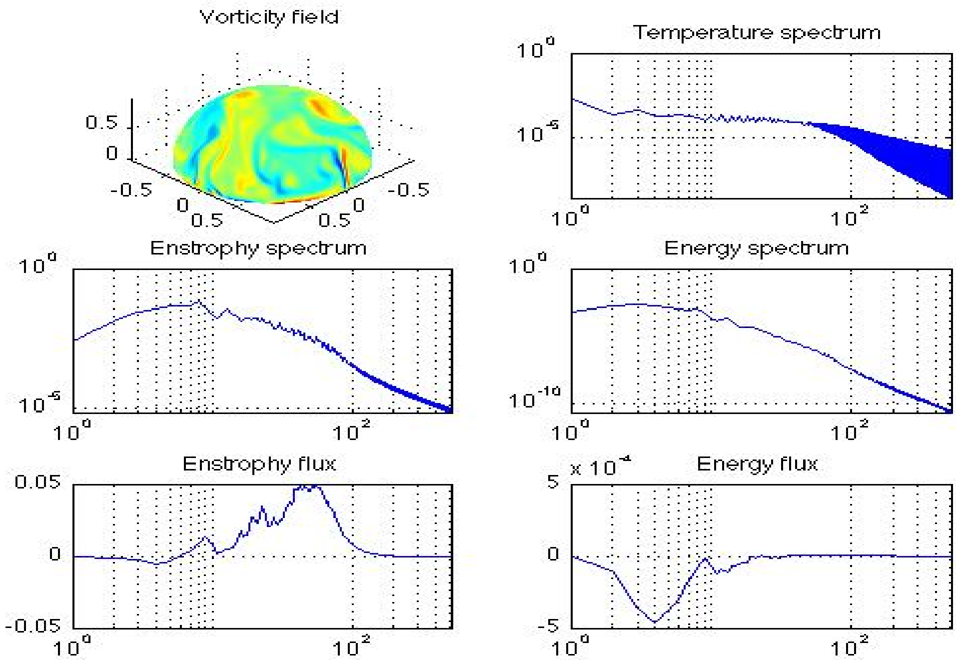

Figure 1: Vorticity field and physical quantities

The expansion coefficients f_l^m are obtained by projection of the function $f(\theta,\phi)$ onto the spherical harmonics $\{Y_l^m\}$ and are used in the same way as Fourier coefficients for computing energy and enstrophy spectra, fluxes, etc. Many simulations have been obtained for different values of the Rayleigh number (a dimensionless number describing natural convection flow; $R_a = \frac{g\beta T}{\nu\alpha}$ where α is the thermal diffusivity) and source parameter c. A snapshot and physical quantities of the simulated bubble with the Rayleigh number equal to 10^7 are given in Figure (1). The slopes observed in the energy and enstrophy spectra are respectively close to k^{-3} and k^{-1} as for usual turbulent fluids. An inverse energy cascade and a direct enstrophy cascade can be observed from the energy and enstrophy fluxes, however we cannot detect any $k^{-5/3}$ in the energy spectrum. The slope in the temperature spectrum is very close to k^{-1} as predicted by the classical theory of turbulence. The vorticity field is very similar to the experimental bubble of soap. We observe plumes rising from the equator zone and merging to form bigger vortices as observed in the physical experiment [1]. Comparisons with the experiments at various Rayleigh numbers as well as simulations of the earth atmosphere will be shown at the conference.

References

[1] F. Seychelles, Y. Amarouchene, M. Bessafi, and H. Kellay. Thermal convection and emergence of isolated vortices in soap bubbles *Phys. Rev. Lett.* 100, 144501, 2008.