High-Order Central ENO Finite-Volume Scheme for MHD on Three-Dimensional Cubed-Sphere Grids

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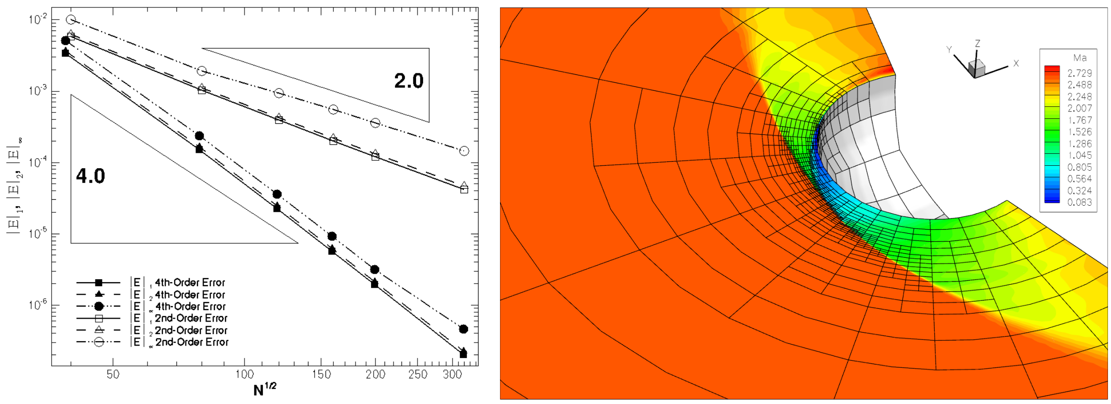
Abstract: A high-order central essentially non-oscillatory (CENO) finite-volume scheme is developed for the compressible ideal magnetohydrodynamics (MHD) equations and applied to space-physics flows on three-dimensional cubed-sphere grids. The CENO scheme is based on a hybrid solution reconstruction procedure that provides high-order accuracy in smooth regions, even for smooth extrema, and non-oscillatory transitions at discontinuities. The scheme is applied in combination with the divergence correction technique proposed by Dedner *et al.* (J. Comput. Phys. 175 (2002) 645-673) to enforce the solenoidal condition for the magnetic field. The cubed-sphere simulation framework represents a flexible design based on a genuine multiblock implementation, leading to high-order accuracy, flux calculation, adaptivity and parallelism that are fully transparent to the boundaries between the six sectors of the cubed-sphere grid. Numerical results to demonstrate the accuracy, robustness and capability of the proposed high-order framework are discussed.

Keywords: Magnetohydrodynamics, High-Order Scheme, Cubed-Sphere Grids.

High-order accurate and efficient computational methods are highly desirable in many fields of computational physics, especially in the study of problems characterized by a wide range of temporal and length scales on which the interesting physics occurs. For global magnetohydrodynamics (MHD) modelling of space-physics problems, high-order methods have the potential to significantly reduce the number of grid elements required to discretize the vast computational domains associated with celestial bodies and the intervening space so as to provide accurate and more affordable predictions of large-scale space-weather phenomena such as solar flares and coronal mass ejections (CMEs). However, the development of accurate and efficient algorithms for space-plasma modelling is challenging due to the intricate nature of the MHD equations which require magnetic monopole free solutions, the presence of collocated regions of smooth and discontinuous solution variations, the complexities encountered in the discretization of spherical domains and the difficulties in prescribing high-order boundary conditions, respectively.

This work considers the development and application of a high-order Godunov-type finite-volume method in a three-dimensional (3D) block-based implementation for prediction of inviscid compressible flows governed by the ideal MHD equations on spherical shell domains. In particular, the proposed algorithm seeks to provide a fourth-order accurate numerical procedure for space-physics problems on 3D cubed-sphere grids [1]. Note that a second-order counterpart has been previously formulated in [2] by employing a multidimensional limited linear reconstruction and a Powell source term approach for controlling the errors in the divergence free condition. Recently,



(a) Grid convergence study for superfast outflow from a rotating cylindrical object.

(b) Predicted Mach number in (x, y)-plane for the 3D MHD bow-shock flow described in the text. The block boundaries are depicted with solid black lines.

Figure 1: Preliminary results to illustrate current capabilities of the proposed algorithm.

the extension to high-order accuracy has been considered and it is described herein. High-order accuracy for MHD equations is obtained in this work by applying a central essentially non-oscillatory (CENO) discretization [3] in combination with the divergence correction technique proposed by Dedner *et al.* [4]. In this approach, a hybrid CENO procedure based on a multidimensional k-exact reconstruction following from a fixed central stencil is applied to each solution variable of a modified system that has the divergence constraint coupled with the conservation laws through the use of a generalized Lagrange multiplier. The hybrid CENO procedure uses an unlimited k-exact reconstruction in smooth regions and reverts to a limited linear reconstruction algorithm in regions deemed as non-smooth or under-resolved by a solution smoothness indicator, thereby providing monotone solutions near discontinuities.

To demonstrate the potential of this approach, two preliminary results are presented. The first one is the radial superfast outflow from a rotating cylindrical object with the magnetic field not aligned to the flow. This two-dimensional problem in nature is suitable for performing grid converge studies for non-field-aligned flows based on flow invariants. The L_1, L_2 and L_∞ error norms depicted in Fig. 1(a) for both second- and fourth-order algorithms demonstrate that the expected order of accuracy is achieved by each scheme. The second problem represents a 3D MHD bow-shock flow around a perfectly conducting sphere solved on solution-adaptive cubed-sphere grids. The predicted Mach number distribution in the Cartesian (x, y) plane obtained after 7 refinement levels on the final refined mesh is shown in Fig. 1(b). This result demonstrates the robustness of the numerical procedure and shows that the complex shock structures arising in front of the sphere are well resolved by the 3D adaptation procedure on cubed-sphere grids.

The proposed solution method provides a high-order accurate numerical procedure for solving compressible MHD flows pertaining to the field of space physics. The final version of the paper will provide details of the combination of the high-order CENO scheme and the divergence cleaning approach for 3D cubed-sphere grids. It will also contain rigorous convergence studies on spherical grids and numerical results for more complex space-physics problems.

References

[1] C. Ronchi, R. Iacono, and P. S. Paolucci. *J. Comput. Phys.*, 124:93–114, 1996.

[2] L. Ivan, H. De Sterck, S. A. Northrup, and C. P. T. Groth. 2011 *AIAA*, Paper 2011-3382.

[3] L. Ivan. PhD thesis, University of Toronto, October 2010.

[4] A. Dedner, F. Kemm, D. Kröner, C.-D. Munz, T. Schnitzer, and M. Wesenberg. *J. Comput. Phys.*, 175:645-673, 2002.