A Two-Dimensional Model of Blood Plasma Flow with Oxygen Transport and Blood Cell Membrane Deformation

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Abstract: Sickle cell disease (SCD) is a genetic disorder that alters the red blood cell (RBC) structure and function such that hemoglobin (Hb) cannot effectively bind and release oxygen. Our novel 2-D computational model represents a fast, time efficient method developed to analyze flow dynamics, O_2 diffusion, and cell deformation in the microcirculation. A finite difference, Crank-Nicholson scheme is used to compute the flow and O_2 concentration, and the level set computational method is used to advect the RBC membrane on a staggered grid. Simulation data indicates a few parameters to be significant in the perturbation of the blood flow and O_2 concentration profiles.

Keywords: microcirculation, sickle cell, red blood cell, finite difference, level set, Crank-Nicholson, blood flow, oxygen diffusion.

1 Introduction

A plausible physical model of blood flow in the capillaries must include the blood plasma dynamics, oxygen diffusion from the red blood cells to the surrounding tissue, and the deformation of the red blood cells' membrane as they are convected downstream. In the case of sickle cell disease, which is a genetic disorder that alters the red blood cell structure and function such that hemoglobin cannot effectively bind and release oxygen, the physical modeling require proper consideration for severe red blood cell membrane deformation. Previous computational models have been designed to study the microcirculation for insight into normal blood flow as well as blood flow disorders such as sickle cell disease. We have developed a novel 2-D computational model that represents a fast, time efficient method to analyze blood plasma dynamics, oxygen diffusion, and cell deformation in the microcirculation. The model uses a finite difference, Crank-Nicholson scheme to compute the blood plasma flow and oxygen concentration profiles, and a level set computational method to advect the red blood cell membrane on a staggered grid. A set of initial and boundary conditions were tested.

2 Problem Statement

The microcirculation plasma flow and oxygen transport are modeled by the following set of governing equations for our system: (1) the continuity equation, (2) incompressible Navier-Stokes equation, and (3) Fick's Law of mass diffusion.

$$\nabla \cdot \underline{v} = 0 \ , \qquad (1)$$

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = -\nabla p + \mu \nabla^2 \underline{v} \ , \tag{2}$$

$$\frac{\partial c}{\partial t} + \nabla \cdot \left(c\underline{v} - D_{ox} \nabla c\right) = R(c) \ , \tag{3}$$

These governing equations were calculated on a marker-and-cell (MAC) staggered grid computational domain, as shown in Figure 1.

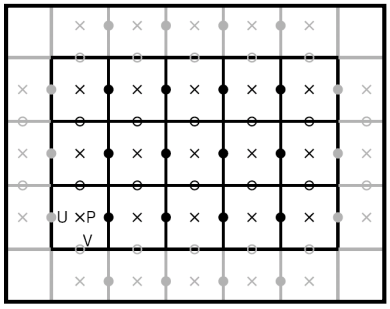

Figure 1 – Staggered MAC Grid

For the advection term in the Navier-Stokes equation, an explicit upwind scheme was used. To ensure computational stability, we apply the CFL condition, which gives us a time step,

$$\Delta t \leq \frac{h}{(u_{max} + v_{max})} \ , \tag{4}$$

for a grid size h, and a maximum x and y components of velocity u_{max} and v_{max}. The movement and deformation of the red cell membrane is driven by the plasma velocity according to the level set propagation rate,

$$F = \frac{u\phi_x + v\phi_y}{\sqrt{\phi_x^2 + \phi_y^2}} \ , \tag{5}$$

where ϕ is the level set function and u and v are the plasma velocity components. Using these methods as the basis of our model, we can simulate the microcirculation applying appropriate boundary and initial conditions to determine the most significant parameters in modeling the sickle microcirculation environment.

3 Conclusion and Future Work

Simulation data shows several important parameters to be significant in predicting the blood plasma flow, red blood cell membrane deformation, and oxygen concentration profiles. Figure 2 illustrates the progressive deformation of the RBC and the stress patterns on the RBC membrane. The variable j represents the stiffness index, which varies in the sickle case in the range $0 \leq j \leq 2$. We notice that the stiffness index plays a large role in the membrane stress and shape as well as the rate of membrane deformation. Specifically, the Hill coefficient, arterial O_2 partial pressure, O_2 partial pressure at 50% Hb saturation, and cell membrane stiffness are significant factors in the microcirculation oxygen concentration and membrane stress/deformation profiles. Results were found to be consistent with those of Le Floch [2] and Secomb [3].

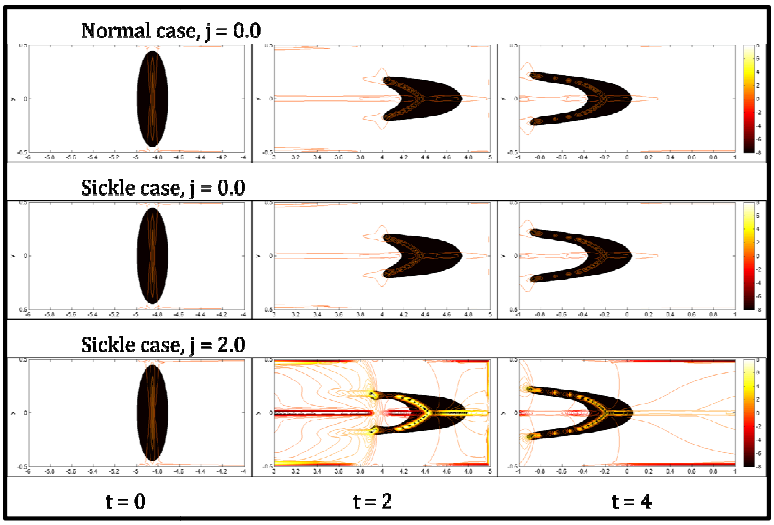

Figure 2 – RBC Deformation and Membrane Stress Profiles

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