New approaches for efficient computation of low Mach number unsteady flows with sound propagation

E.Shima*,
Corresponding author: shima.eiji@jaxa.jp

* Japan Aerospace Exploration Agency, JAPAN

Abstract: This paper deals with the new method to compute very low mach number flow and sound propagation at same time. The new method is shown effective both steady and unsteady flows.

Keywords: Numerical Algorithms, Computational Fluid Dynamics, Low Mach Number, Aeroacoustics.

1 Introduction

In some applications, such as combustion in liquid rocket engine combustor, sound in unsteady low Mach number flow plays significant role. The authors recently proposed all speed numerical flux function, named SLAU[1], that can compute from very low (<0.1) to high (>10) flows accurately without parameter adjustment. It has been shown it can compute low Mach number flows and sound at the same time by explicit time integration. This paper deals with new efficient implicit time integration methods for SLAU in low Mach number regime.

2 Formutation

The three point backward Euler method for finite volume method for Navier-Stokes equation in delta form forms linear system as follows introducing the entropy variable **W** as working variables.

$$\left[1.5\frac{\mathbf{I}}{\Delta t}+\frac{1}{V_i}\sum_j s_{i,j}\left(\frac{\mathbf{D}}{2}+\tilde{\mu}\right)_{i,j}\right]\Delta\mathbf{W}_i-\frac{1}{V_i}\sum_j s_{i,j}\left(\frac{\hat{\mathbf{B}}_{j,i}+\mathbf{D}}{2}+\tilde{\mu}\mathbf{I}\right)\Delta\mathbf{W}_j=-\mathbf{M}_i\mathbf{H}_i^k$$

where **M** is the transformation matrix from conservative to entropy variabel and **H** contains explict residual and difference between inner iterations when Newton method is considered. By this transformation, Jacobian **B** is simplified. Our effort will be consentrated on solving this linear system. For computational efficiency, implicit numerical dissipation **D** is assumed to have diagonal form as;

$$\mathbf{D}=diag\lfloor d_p,d_u,d_u,d_u,d_s\rfloor$$

where d_p,d_u,d_s are the disspations for pressure, velocity and entropy. Approximate form of the dissipation corresponds to that of SLAU can be written as follows with the consideration for avoiding zero dissipation at stagnation;

$$d_p=|V_n|+c\quad d_u=d_s=|V_n|+(1-\chi')c\quad \chi'=(1-\hat{M}')^2\quad \hat{M}'=\min\left(1.0,\sqrt{\frac{u^2+v^2+w^2}{\bar{c}^2}+Mc^2}\right)$$

where V_n, c and Mc are convective velocity normal to cell face, speed of sound and cutoff Mach number. This set of implicit numerical dissipations does not make positive definite numerical Jacobians which is necessary for use of SGS. Thus this matrix system has to be solved by modern algorism such as GMRES. FGRMRES(k)[3], where k is the dimension of Krylov subspace, is used in this study. On the other hand, following modification ensures positive definiteness;

$$d_p=|V_n|+c/(1-\chi')\quad d_u=d_s=|V_n|+(1-\chi')c$$

The SGS iteration using these dissipations is named as TC-PGS(Time Consistent Preconditioned GS)1, because it can be shown that it has close relation with low Mach number preconditioning of Weiss&Smith.[2] TC-PGS1 is also used as the matrix pre-conditioner for FGMRES.

3 Results

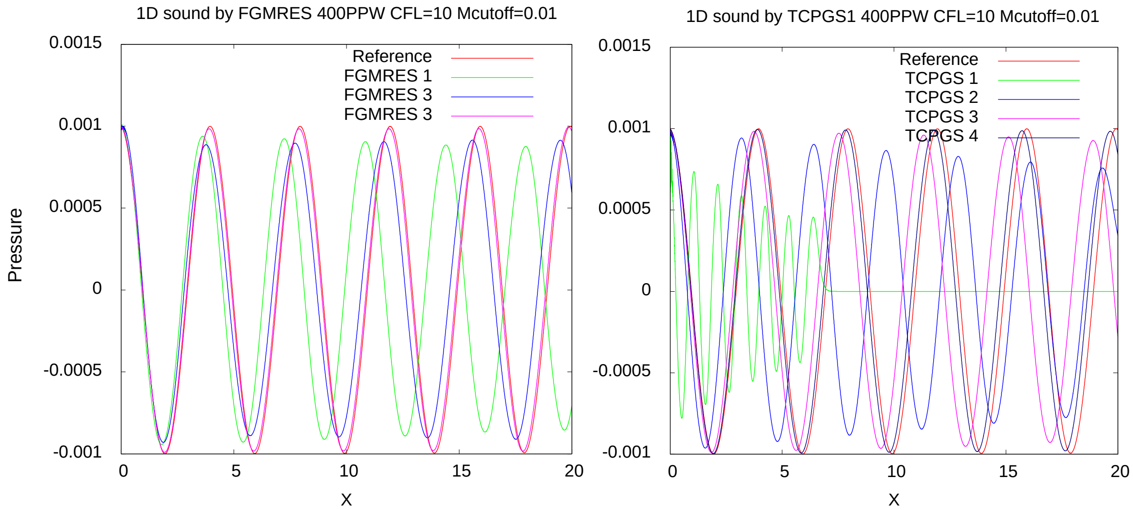
Numerical experiments, such as computations of unsteady separated flow around a cylinder, showed that;

- Rather small *k* of FGMRES gave good overall efficiency and FGMRES(4) is chosen.
- FGMRES(4) and TC-PGS1 showed comparable efficiency, when only fluid motion is of interest. Also both have similar performance to time derivative preconditioning for steady flows

As an example of sound propagation, 1D pressure propagation with Mc=0.01, which is suitable for low Mach number flows, is computed by TC-PGS1 and FGMRES(4) with various Newton iterations. Cases are shown in Table 1, and results of FGMRES(4) and TC-PGS1 are shown in Fig.1. For acceptable accuracy, FGMRES(4) is over 10 times more efficient than TC-PGS1.

Table 1.

CASE	METHOD	Non-linear Iteration	Relative CPU Time
TCPGS 1	TCPGS1	8	1.28
TCPGS 2	TCPGS1	100	16.66
TCPGS 3	TCPGS1	200	33.69
TCPGS 4	TCPGS1	300	50.89
FGMRES 1	FGMRES(4,1)	2	1.00
FGMRES 2	FGMRES(4,1)	4	1.95
FGMRES 3	FGMRES(4,1)	8	3.91



(a) (b)

Fig.2 Pressure distribution in one dimensional sound propagation. Results by FGMRES(4.1) in the left(a) and by TC-PGS1 in the right(b).

3 Conclusion

Two methods for unsteady compressible flow in low Mach number was shown. TC-PGS1 is essentially a implicit time integration with low Mach number preconditioning in simple form, and also FGMRES using TC-PGS1 as a matrix pre-conditioner is shown. When only flow dynamics is in interest, simpler TC-PGS1 which is simpler and has comparable performance to FGMRES will be beneficial. When sound is computed at the same time, FGMRES is over 10 times more efficient.

References

[1] E.Shima and K.Kitamura, “Parameter-Free Simple Low-Dissipation AUSM-Family Scheme for All Speeds”, AIAA Journal, vol.49 ,no.8 (2011)

[2] J.M. Weiss and W.A. Smith, Preconditioning Applied to Variable and Constant Density Flows, AIAA J., Vol. 33, No.11, pp.2050-2057 (1995)

[3] Y.Saad, “A flexible inner-outer preconditioned GMRES algorithm”, SIAM Journal on Scientific Computing, (1993) J. Doe. *Important book title: A complete work.* ACME(2012)