A discontinuous Galerkin based multiscale method for compressible multiphase flow

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Abstract: In this study, a numerical method for the simulation of compressible two-phase flows is presented. The multiscale approach consists of several components: a discontinuous Galerkin (DG) solver for the macroscopic scales of the flow, a micro-scale solver at the interface, a level-set interface tracking formalism as well as adaptive mesh refinement near the interface. The method is demonstrated for a droplet at equilibrium described by the Euler equations including surface tension as well as a shock-droplet interaction without surface tension.

Keywords: DNS, Discont. Galerkin, Multiscale, Compressible multiphase flow.

1 Introduction

The numerical simulation of compressible multiphase flow introduces additional difficulties compared to the incompressible treatment that is often found in two-phase numerical solvers today. Two elements are crucial: A method that allows to define the geometry and the temporal evolution of the interface between the two phases (in this study, we employ a level-set method) as well as a numerical strategy to treat the discontinuous nature of the interface as well as the related physical processes such as surface tension or phase change. In the compressible case, this includes different equations of state (EOS) to both sides of the interface. In principle, the interface region can be approximated by a smooth transition that can be handled by the numerical scheme (diffuse interface approach). As an alternative, we use a sharp interface multiscale approach, where the numerical scheme only resolves the macroscopic scales of the flow and allows discontinuous states at the interface position, where jump conditions have to be provided by an additional solver for the micro-scale. Many micro-scale solvers are conceivable, in the present study, we rely on Riemann-type solvers [1], [2]. The approach is suitable for general EOS.

2 Sharp-interface multiscale approach

The macro-scale solver for the bulk phases of the flow uses a DG spectral element method [3], formulated for hexahedral elements. The solution as well as the fluxes are approximated by a tensor-product basis based on Lagrange Polynomials. Volume and surface integrals are approximated by Gauss-Legendre-Lobatto quadrature. As no continuity constraint is enforced

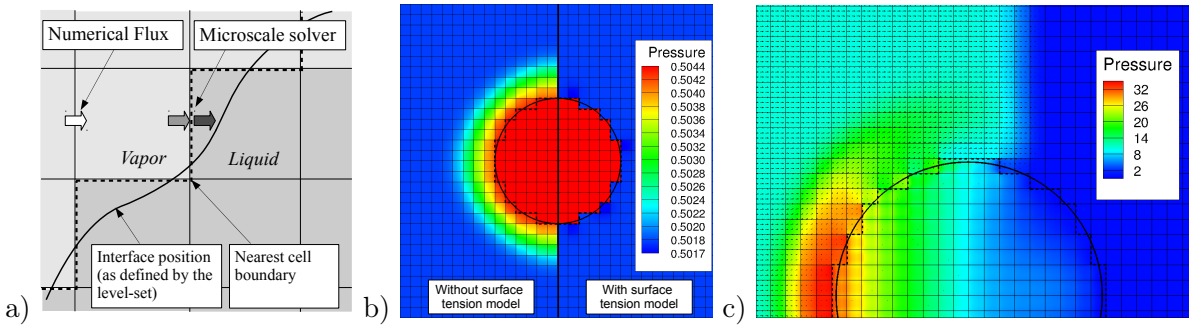

Figure 1: a) Pressure field of a droplet with phase change. Comparison with surface tension model non-active (left) and active (right). b) Pressure field of a shock-droplet interaction.

between the elements, the flux function at cell boundaries is replaced by a numerical flux. In the bulk phases, standard Riemann solvers are used. Near the phase interface, the jump in fluid quantities is assumed to coincide with the nearest element boundary, where the Riemann solver is replaced by the micro-scale solver (Fig. 1 a). Instead of evaluating a single numerical flux, the micro-scale solver provides discontinuous fluxes, which are applied on the liquid and gaseous side respectively. In order to reduce errors related to the discrepancy between the actual interface (the level-set zero) and the nearest element boundary, an adaptive grid refinement is devised for elements in a neighborhood of the interface. The scheme allows high orders of accuracy with subcell resolution, which is particularly valuable for the calculation of interface curvature that can be obtained from derivation of the ansatz polynomials for the level-set variable. The model for surface tension is included in the micro-scale solution and takes effect in the macroscale via the numerical fluxes, thereby eliminating the need for source terms to apply the surface force.

3 Results

We present two 3D test cases to demonstrate the different aspects of the numerical approach. AMR has not been used in these simulations but will be included in the final paper. The first test incorporates surface tension: a droplet is initialized in such a way that the higher inside pressure is at equilibrium with surface tension. The result is obtained for the full Euler equations with Tait and ideal gas EOS. Fig. 1 b) compares the equilibrium state with surface tension and a non-equilibrium state without it (100 iterations). Diameter and shape remain constant, the pressure jump is kept sharp, while fluctuations near the interface are well-controlled. The second test (Fig. 1 c) is a shock-droplet interaction. The interface remains smooth despite the staircase pattern of the interface aligned with the element boundaries. The effect of a higher propagation speed inside the droplet and wave reflection on the interface can be observed.

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