Integrated Adaptive Numerical Method for Transient Two-phase Flow in Heterogeneous Porous Media

C.C. Chueh[1], W. Bangerth[2] and N. Djilali[1]
Corresponding author: ndjilali@uvic.ca

[1]Institute for Integrated Energy Systems & Dept. of Mech. Eng., Univ. of Victoria, Canada
[2] Department of Mathematics, Texas A&M University, USA

Abstract: An interconnected set of algorithms is presented for the simulation of two-phase flow in porous media achieving more than two orders of magnitudes acceleration. The accuracy and efficiency of the approach is demonstrated through 2D and 3D numerical experiments for a range of two-phase porous media problems involving single cracks, heterogeneous permeability, and hydrophobic and hydrophilic media

Keywords: Adaptive refinement; Stabilized finite element method; Operator splitting; Preconditioning; Two-phase flow; Porous media; Fuel cells.

1 Introduction

Multiphase flow in porous media are ubiquitous in applications ranging from flow of groundwater and secondary oil recovery, to geo-sequestration of carbon dioxide and transport processes in fuel cell electrodes [1]. The simulation of such flows poses a number of physical and computational modelling challenges. In this work we focus on the key issues of enhancing computational efficiency of finite element methods while retaining accuracy. An computational framework integrating the following features is presented : (i) higher order spatial discretization yielding the same accuracy at smaller computational cost, and incorporating stabilization mechanisms for hyperbolic problems such as those representing multiphase flow; (ii) adaptive mesh refinement that vastly reduces the number of cells required to resolve the flow field; (iii) adaptive time stepping that allows the use of large time step limited solely by the physical time scale rather than numerical stability; (iv) an operator splitting algorithm allowing the use of efficient solvers; (v) an efficient solver and preconditioning method that accelerate the solution of the linear problem.

These methods were implemented and integrated using the `deal.II` open source finite element library [2].

2 Problem Statement

Viscous effect dominated two-phase flow of an immiscible, incompressible fluid in a porous medium is governed by:

$$\mathbf{u}_t = -\mathbf{K}\lambda_t(S)\nabla p \tag{1}$$

$$\nabla \cdot \mathbf{u}_t = q \tag{2}$$

$$\epsilon \frac{\partial S}{\partial t} + \nabla \cdot (\mathbf{u}_t F(S)) = 0 \tag{3}$$

where S is the saturation (volume fraction) of the second (wetting) phase, p is the pressure, $\mathbf{K}$ is the permeability tensor, λ_t is the total mobility, ϵ is the porosity, F is the fractional flow of the wetting phase, q is the source term and $\mathbf{u}_t$ is the total velocity. Equations (1)–(3) are augmented by relations for the total mobility, fractional flow of the wetting phase and total velocity, by constitutive relations for the relative permeabilities, and by initial conditions for the saturation and boundary conditions for the pressure.

3 Numerical method

Adaptive operator splitting and time stepping: The time stepping schemes most commonly used to solve equations of the kind of (1)–(3) are of IMPES (*implicit pressure, explicit saturation*) type. Noting that the pressure and velocity fields depend only weakly on saturation, a net reduction in computing time can be achieved by rebalancing the computing efforts between the saturation and the pressure/velocity system. An operator splitting approach was implemented to achieve this by solving for the saturation at every time step, and only updating the velocity and pressure whenever necessary at specific "macro time-steps". A key feature is letting the length of the macro time-steps vary *adaptively* as a function of the saturation rate of changes.

Spatial discretization and adaptive mesh refinement: The IMPES scheme requires the separate solution of velocity/pressure and saturation equations. Using a finite element method, we discretize these on the same mesh composed of quadrilaterals or hexahedra, using continuous Q_2 elements for the velocity and Q_1 elements for the pressure. Q_1 finite elements are used for the saturation equation. A shock-type adaptive refinement method described in [3] is used.

Artificial diffusion stabilization of the saturation equation: The Q_1 elements for the saturation equation require stabilization. To avoid smearing of sharp fronts and grid-orientation difficulties associated with classical artificial diffusion, we use the artificial diffusion term proposed by Guermond and Pasquetti [4], which acts primarily in the vicinity of discontinuities and therefore allows a higher degree of accuracy. An explicit Euler time stepping method is used to avoid the difficulty of dealing with the non-linear stabilization term.

4 Results

2D and 3D simulations for a range of two-phase porous media problems involving single cracks, heterogeneous permeability, and hydrophobic and hydrophilic media involving capillary transport were performed demonstrating improved accuracy and up to two orders of magnitude acceleration compared to traditional techniques. Figure 1 shows an example of the initial evolution of the saturation field and associated mesh adaptation in a hydrophobic porous medium relevant to capillary flows encountered in the porous electrode of a polymer electrolyte fuel cell, where water is generated as a result of the electrochemical reaction [1].

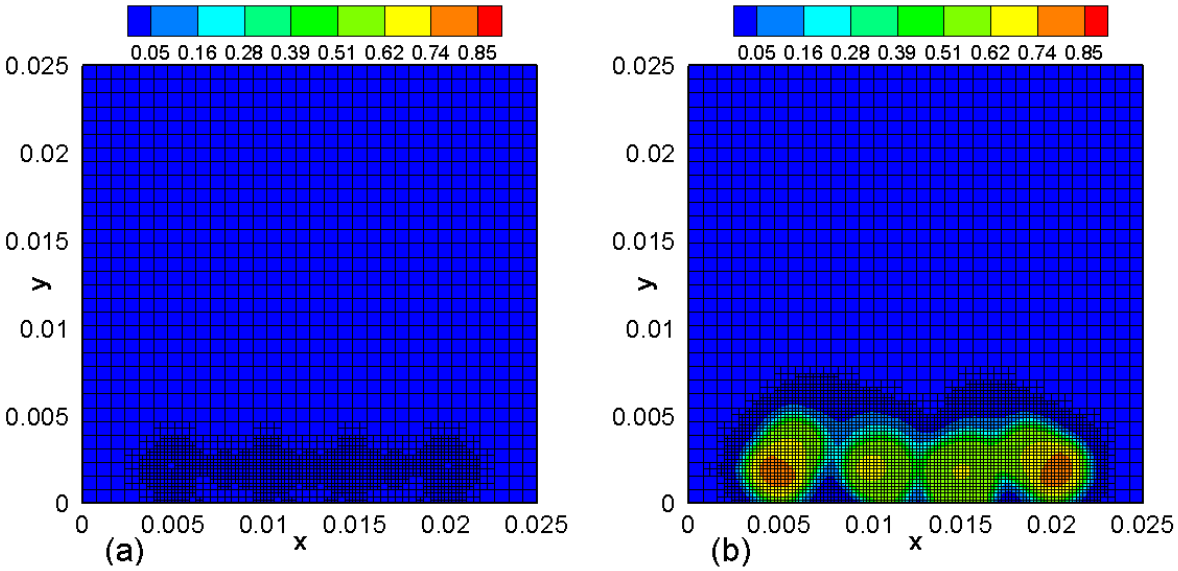

Figure 1: Time evolution of saturation contours and corresponding adaptive grids (a) at $t = 3.71 \times 10^{-5}$ *sec* with 2452 cells and 26126 DoFs (b) at $t = 2.4 \times 10^{-2}$ *sec* with 4105 cells and 42442 DoFs .

5 Conclusion and Future Work

The effectiveness and accuracy of the proposed methodology was demonstrated for a range of two-phase flow problems. The functionality and usefulness of the integrated method can be further enhanced by extending the physical modelling and simulation capabilities to three-phase flows in heterogeneous media, and to coupled processes involving electro-chemical reactions

References

[1] N. Djilali and P.C. Sui. Transport phenomena in fuel cells: from microscale to macroscale, *Int. J. Comp. Fluid Dynamics* 22:115-133, 2008.

[2] W. Bangerth, R. Hartmann and G. Kanschat. deal.II – a General Purpose Object Oriented Finite Element Library, *ACM Trans. Math. Softw.*, 33:24/1-24/27, 2007.

[3] C.C. Chueh, M. Secanell, W. Bangerth and N. Djilali. Multi-level adaptive simulation of transient two-phase flow in heterogeneous porous media, *Comp. & Fluids*, 39:1585-1596, 2010.

[4] J.L. Guermond and R. Pasquetti. Entropy-based nonlinear viscosity for Fourier approximations of conservation laws, *Comptes Rendus Mathematique*, 346:801-806, 2008.