On the High-Order Multidimensional Gas-Kinetic Scheme

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Abstract: The newly developed high-order-accurate multidimensional gas-kinetic scheme is further investigated, including the benefit of the consideration of tangential slopes in the flux function at a cell interface, and the application of the scheme in turbulence simulation. The present study shows that in despite of increasing of computational cost, the multidimensional scheme can evidently improve the accuracy for both inviscid and viscous flows when compared to the directional splitting one. The numerical simulation of the compressible turbulence with the high-order multidimensional gas-kinetic scheme shows better performance than the existing second-order gas-kinetic BGK method.

Keywords: High-Order Gas-Kinetic Scheme, Multidimensional Method, Compressible Turbulence

The foundation for the development of modern compressible flow solver is the Riemann solution of the inviscid Euler equations. However, due to the lack of a multidimensional Riemann solution, it is also a great challenge to construct a genuinely multidimensional scheme. An alternative approach to develop a CFD scheme is based on the gas-kinetic theory, such as the multidimensional BGK-NS flow solver, which has shown good performance in many flow fields. The success comes from the fact that the kinetic equation has the mechanism to accurately describe the gas evolution starting from an initial discontinuous data, including the inherent multidimensional characteristics of the particle transport. Recently, through the high-order expansion of equilibrium distribution function the high-order multidimensional gas-kinetic BGK scheme has been successfully developed, which puts a new way to construct high-order-accurate truly multidimensional scheme (HBGK) for compressible flows.

In the present study, the difference in performance among the multidimensional (MD) third-order gas-kinetic BGK scheme, the corresponding quasi-one-dimensional (Q1D) extension and the directional (DS) scheme are investigated for both viscous and inviscid flows. The isentropic vortex problem is considered, with initial flow field $\rho=1,\ p=1, u=v=1$ and perturbation $(\delta u,\delta v)=\varepsilon\exp[(1-r^2)(-y,x)/2]$, $\delta T=(\gamma-1)\varepsilon^2/(2\gamma)\exp[1-r^2]$, $\delta S=0$ and $\varepsilon=5/(2\pi)$. The computational domain is $[-5,5]\times[-5,5]$ divided by uniform cells. Periodic boundary conditions are adopted. The computed error variations with different cell sizes are shown in figure 1, where the benefit of the including of tangential slopes in the flux function at a cell interface is evident.

The effect of the high-order reconstruction for the multidimensional gas-kinetic scheme is investigated with the subsonic flat plate boundary layer flow. The numerical simulations show that for small transverse stretching rate of the computational cell, such as $\eta = 1.2$, the second-order reconstruction can give very good results. For example, the transverse velocity upstream can be accurately predicted with only four cells. However, if increasing the stretching rate, the third-order reconstruction can yield better transverse velocity profiles, especially near the outer edge of the boundary layer.

The three-dimensional scheme is then developed and applied into the numerical simulation of compressible turbulence. Several typical cases, such as the isotropic homogeneous turbulence and compressible turbulence in a plat channel, are tested and the results show better performance of HBGK than the existing second-order gas-kinetic BGK method.

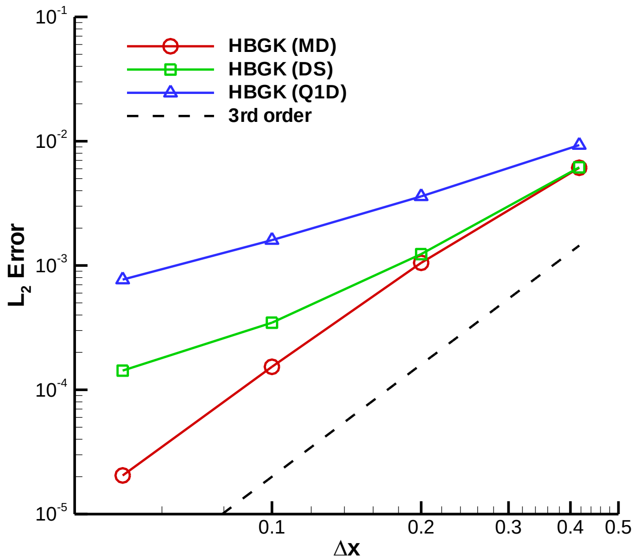

Figure 1: Errors in density vs. cell size for isentropic vortex problem

References

[1] K. Xu, A gas-kinetic BGK scheme for the Navier-Stokes equations, and its connection with artificial dissipation and Godunov method. *J. Comput. Phys*. **171**, 289 (2001)

[2] Q.B. Li, K. Xu, S. Fu, A high-order gas-kinetic Navier-Stokes flow solver, *J. Comput. Phys.* **229**, 6715-6731 (2010).

[3] Q.B. Li, K. Xu, S. Fu, A new high-order multidimensional scheme, Computational Fluid Dynamics 2010: Proceedings of the Sixth International Conference on Computational Fluid Dynamics, ICCFD6, St Petersburg, Russia.