Analysis of high-order residual-based dissipation for unsteady compressible flows

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Abstract: A comprehensive study of the numerical properties of high-order residual-based dissipation terms for unsteady compressible flows leads to the design of well-behaved, low dissipative schemes of third-, fifth- and seventh-order accuracy. The dissipation and dispersion properties of the schemes are then evaluated theoreticaly, through Fourier space analysis, and numerically, through selected test cases including the inviscid Taylor-Green Vortex flow.

Keywords: Residual-based scheme, High-order, Unsteady, Dissipation.

1 Introduction

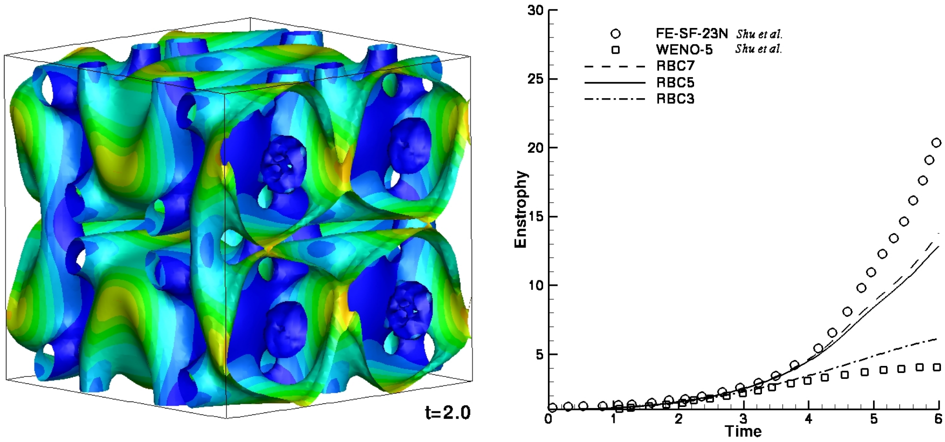
Classical methods for calculating compressible flows on a structured mesh rely on a directional approach in which space derivatives are approximated independently direction by direction. In the present work, we study compact approximations that provide a high accuracy not for each space derivatives treated apart but for the complete residual r, *i.e.* the sum of all the terms in the governing equations. For unsteady problems, r also includes the time derivative. Schemes of this type are said to be Residual-Based Compact (RBC) and have been developed in the last ten years ([1, 2] for instance) with application to realistic flow configurations in aerodynamics and aeroacoustics. A special feature of the RBC scheme is the structure of its numerical dissipation also constructed from the complete residual r. This dissipation gives interesting properties to the RBC schemes that have not been fully analysed for unsteady problems so far. Here, we present an in-detailed study of the residual-based dissipation involved in high-order RBC schemes for the unsteady Euler equations. This study allows a better understanding of the dissipation mechanism and leads to some improvements of the existing RBC schemes. Given the importance of numerical dissipation in Computational Fluid Dynamics, it is also hoped that the present work could help the development of other classes of high-order schemes.

2 Analysis of the RBC dissipation

For unsteady problems the numerical dissipation of RBC schemes $\tilde{d}$ is an approximation of the following operator:

$$d = \frac{\delta x}{2}\left[\Phi_1 r\right]_x + \frac{\delta y}{2}\left[\Phi_2 r\right]_y + \frac{\delta z}{2}\left[\Phi_3 r\right]_z$$

where Φ_i, $i = 1, 2, 3$ are matrices given in [1]. For an exact solution of the unsteady problem, d is null and $\tilde{d}$ is consistent with a higher order error term. In our work, we precisely analyse this term since it governs the scheme dissipative properties: indeed, numerical dissipation is not really consistent with the operator d, but with the first error term produced by the discretisation of d. Based on this approach third-, fifth- and seventh-order accuracy unsteady RBC schemes are designed taking into account a necessary condition for stability. A study of the numerical spectra associated with the RBC spatial discretizations is carried out and strengthens the previous results. Finally, a detailed spectral analysis of the schemes is performed in order to quantify dissipative and dispersive errors induced by their spatial discretizations.



(a) (b)

Figure 1: (a) Iso-surface of the Q-criterion colored by the kinetic energy at $t = 2.0$, RBC5 scheme (64^3 mesh). (b) Time evolution of the normalized enstrophy for RBC3, RBC5, RBC7, WENO-5 and a Fourier Collocation method with a sharp-cutoff filter (FE-SF-23N [3]).

A simple linear advection problem (transport of a sinus wave) further confirms the precedings conclusions and analysis. This test case allows to demonstrate the low-dissipation, low-dispersion properties of the schemes. Then the schemes are applied to the computation of the converging cylindrical shock showing the good shock capturing properties of the schemes. Finally the RBC schemes are applied to the inviscid Taylor-Green Vortex flow [3]. This problem has been proven to be an excellent testing bench to study the resolvability properties of high-order schemes in view of their subsequent application to fine-scale turbulence simulations. Preliminary results for this problem are shown in Fig. 1. The high-order schemes under analysis provide a good representation of integral quantities such as the total enstrophy over the computational domain. In the final paper we will provide details of the theoretical and computational dissipation analysis along with a full description of the test cases.

References

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