Dissipation of upwind schemes at high wave numbers

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Abstract: A modification of the Roe scheme enhanced for low Mach numbers is discussed. It improves the dissipation of kinetic energy at the highest resolved wave numbers in a low Mach number test case of decaying isotropic turbulence. No conflict is observed between the reduced dissipation and the accuracy or stability of the scheme in any of the investigated test cases ranging from low Mach number potential flow to hypersonic viscous flow around a cylinder. First steps to modify AUSM type schemes are discussed as well.

Keywords: Riemann Solvers, Upwind schemes, Dissipation.

1 Introduction

When approximate Riemann solvers were introduced into Godunov type schemes about three decades ago an efficient way of simulating gas dynamic flows was found enabling to capture shocks within a few grid cells. In the following years many different approximate schemes were created and successfully improved to satisfy the entropy condition and handle the carbuncle phenomenon. Focusing on the schemes for one perfect gas, the development of the numerical flux functions was then dominated for many years by modifications to enable accurate results at low Mach numbers without loosing the stable and accurate resolution of high Mach number flow features. With the increasing computer power of recent years, the possibility to investigate time resolved high Mach number turbulent flow fields via Large-Eddy or Detached Eddy Simulations exhibits a new challenge for upwind schemes. The energy conservation properties of the numerical scheme are damaged by numerical dissipation in the range of then higher resolved wave number regime.

Thornber suggests a modification of the MUSCL reconstruction step [1], which changes the response of the flux function by reducing the velocity jump between the left and right states with decreasing Mach number. Beside a positive influence in the high wave number regime, it even improves the low Mach number ability of many Riemann solvers, as long as they are not already modified to resolve low Mach number flows. Combining it with low Mach number versions, the dissipation of the resulting scheme becomes too low. Based on the low Mach number modification of the Roe scheme proposed by Rieper [2], we investigate a different approach: reducing the dissipation within the Riemann solver in order to improve the solution for high wave numbers.

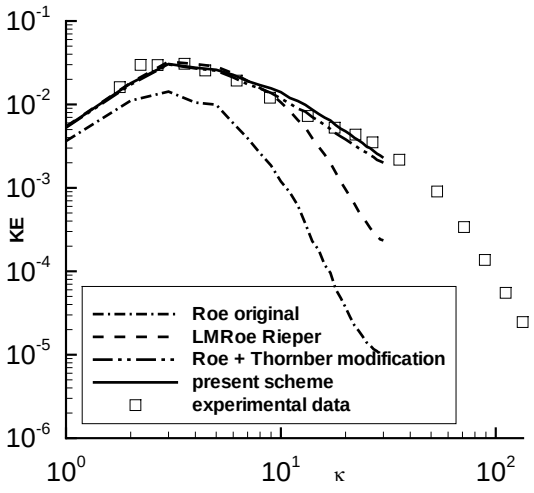

Figure 1: Turbulent kinetic energy spectrum at t=0.87s in the case of decaying isotropic turbulence; experimental data taken from Comte-Bellot et al., JFM 1971.

2 Modification and Results

Rieper reduces the part of the dissipation in the Roe scheme, which is generated by the jump of the normal velocity component and is associated with the acoustic waves. A similar switch applied to the dissipation generated by the jump of the tangential velocity components results in an improved scheme for high wave numbers in low Mach number flows. Decaying isotropic turbulence is investigated on an equidistant cartesian grid. The distribution of kinetic energy over the wave number is depicted in figure 1. The modified version clearly improves the numerical result and compares well with the experimental values. Although the starting point is similar to the modification suggested by Thornber, the modified Roe scheme suggested here behaves more stable especially on unstructured grids. Including the shock-fix idea from Liou [3], the scheme was tested in a number of cases ranging from low Mach number potential flow to hypersonic viscous flow around a cylinder. The reduced dissipation of the scheme was not a problem in any of these cases.

Motivated by the observations made during modifying the Roe scheme, an idea about changing AUSM type schemes is tested as well. A promising effect can be reported, but not as beneficial as in the case of the Roe solver. Nevertheless, the development towards improved upwind schemes will be continued.

References

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