Unresolved Problems by Shock Capturing

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Abstract: Overheating problem, first observed by von Neumann and later studied extensively by Noh [1] using both Eulerian and Lagrangian formulations, remains to be one of unsolved problems by shock capturing. It is historically well known to occur when a flow is under compression, such as when a shock wave is brought to rest and reflected from a wall or collides with another. We identify that the overheating phenomenon can also appear in a smooth flow undergoing only rarefaction. This is in contrary to one's intuition expecting a decrease in internal energy. The excessive increase in temperature is insensitive to refining mesh size. We found that the root lies in the inaccurate treatment of stagnant contact discontinuity of diminishing strength by the shock capturing method.

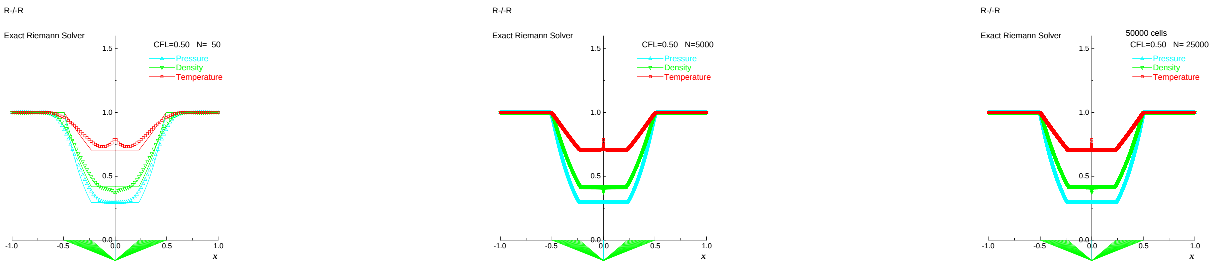
Keywords: Numerical Algorithms, Overheating Problem.

1 Introduction

With the enormous advances in numerical algorithms and computer technology, solving a complex three dimensional problems is nowaday rather common with a desktop computer or local clusters. It is easy to conclude that the basic algorithmic development is no longer necessary, except the continuing need for physical modelling and mesh generation. On the contrary, the author argues that there are still some fundamental problems that are yet to be settled by the *shock capturing approach in the Eulerian framework*. For this paper, our objectives are: (1) to revisit some longstanding (open) problems that still require much needed attention, based on a wide class of modern methods, (2) identify new problems, and (3) to clarify the roots of difficulty and suggest a cure for it. Preliminary results suggest that reformulating the mathematical system and flux functions holds promises for unlocking the mystery.

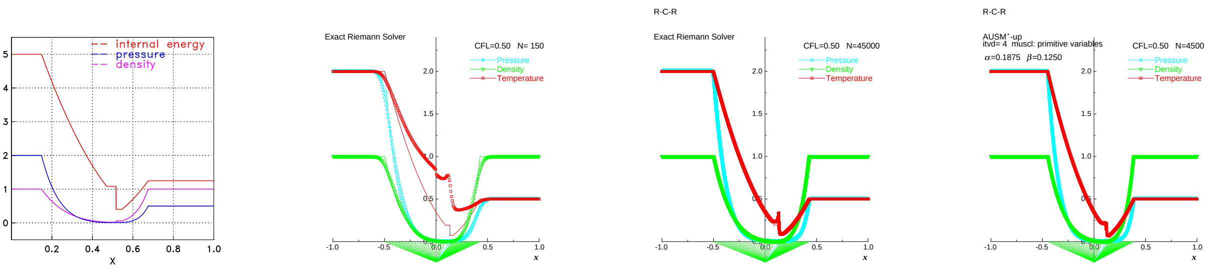
2 Overheating problem

First we consider a usual Riemann problem with the following initial left and right states: $(p, \rho, M)_L = (1.0, 1.0, -0.8)$ and $(p, \rho, M)_R = (1.0, 1.0, 0.8)$. That is, two sections of fluid begin to more away from each other, thus creating a low pressure, low density and low temperature region between them. This region grows with time, possibly leading to a vacuum condition if the separating velocity is sufficiently large. This flow is composed of two rarefaction waves that are connected by an intermediate state, a contact discontinuity of diminished strength in this case. The correct solution is presented in Figure 1, denoted by the solid lines, showing a constant state in the middle. Included in this figure is the solution by the "exact" Riemann solver – the Godunov method. Aside from the inaccurate representation of the head of the rarefaction region, most notable is an excessive temperature in the mid-section, even though there is no heat added. Intuition would suggest that cooling down, instead of heating up should happen at the center where the receding began. Hence, we call it the "overheating" problem. This overheating, however, is different from the historically well-known one in which the excessive heat occurs when the flow is under compressive environment, such as a flow colliding with a wall or an opposing flow. Clearly, this error is the creation of a numerical method. It must be pointed out that this difficulty does



(a) 100 cells. (b) 10000 cells. (c) 50000 cells.

Figure 1: Solution profiles given by the exact and Godunov methods.



(a) Exact solution. (b) 300 cells, Godunov. (c) 100,000 cells, Godunov. (d) 10,000 cells, present.

Figure 2: Comparison of exact solution and numerical solutions with the Godunov and present methods on different meshes.

not stay with the Godunov method only; it exists in every upwind method that the author has tested. The phenomenon persists by all possible operational variations, such as grid size, time step, high order interpolation and limiters, time-stepping procedures, etc. Figure 1 gives the results of using 50,000 cels; while the agreement with the exact solution is greatly improved, the "overheating" is nearly as pervasive as before, with the maximum error remaining unchanged.

It must be reminded that if the separating velocity is increased, maintaining positivity for pressure, density and temperature becomes an important consideration in order to continue solution. Now if a jump in temperature between the two fluid sections is imposed initially, as given by $(p, \rho, M)_L = (2, 1.0, -2.5)$ and $(p, \rho, M)_R = (0.5, 1.0, 2.5)$. Its exact solution is shown in Fig. 2 as a reference. Noticing that two rarefactions, differing from the previous problem, have developed a contact of finite strength, albeit tiny, after $t > 0$. The ability required for capturing the contact discontinuity makes the matter more exasperating, as evident in Fig. 2, respectively on a coarse and extremely fine mesh. Similar situation as the first Riemann problem happens here as well. The coarse grid solution simply grossly overpredicts the temperature distribution and gives a hint of overheating at the contact discontinuity. Even after employing 100,000 cells, Fig. 2 confirms that while the region of overheating shrinks by refining mesh size, the overheating level at the contact discontinuity does not appear to change at all.

To eradicate this overheating problem, a study into the root of the problem has started, with related work reported in Ref. [2]. Currently, some plausible mechanisms have been identified and encouraging preliminary results are obtained, as shown in Fig. 2, where the temperature level now matches with the theoretical value. Even though convergence is proved, it is still notoriously inefficient – using 10,000 cells in 1D! It is expected that more concrete findings will be available at the time of conference.

References

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