Fast Iterative Methods for Navier-Stokes Equations with a SST Turbulence Model and Dual Time Steps

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Abstract: The convergence of a Runge-Kutta scheme for the compressible Navier-Stokes equations with a k-ω/SST turbulence model is accelerated by using an implicit smoother. This enables using high CFL numbers greater than 1000 for both the N-S equations and the turbulence model equations. We also use the same preconditioner to solve time dependent problems using a dual time step procedure. We present solutions for flows about an airfoil.

Keywords: Computational Fluid Dynamics, Turbulence, Dual Time Steps, Preconditioning.

Description of Method

The compressible Navier-Stokes equations is a system of mixed type that also includes discontinuities across shocks. In many applications one is interested in the steady state of this system. However, because of the complicated form of these equations it is customary to solve them in (pseudo) time until a steady state is achieved. An explicit scheme requires ten of thousands of iterations to reach a steady state. This has been reduced to several hundred iterations by using implicit schemes and/or multigrid and residual smoothing. Nevertheless, for many applications this still requires an extended computation for complex three dimensional flows. In [1,2] an explicit Runge-Kutta scheme was used coupled with multigrid and a preconditioner based on a first order upwind scheme, coupled with a Gauss-Seidel approximate solver. This reduced the computation time by a factor of 3-4. However, all the examples used an algebraic Baldwin-Lomax turbulence model. In this paper we extend the results to a two equation k-ω/SST turbulence model. This is accomplished by using the same combination of a Runge-Kutta scheme with an implicit treatment of the forcing function and a first order accurate preconditioner using a separate Jacobian for the turbulence equations which are decoupled from the fluid equations. The result allows time steps of the order of several hundred CFL for **both** the fluid and turbulence equations. The complete problem requires only a few iterations to reduce the residual several orders of magnitude. The scheme for the turbulence equations is given by

$$\left[I + \varepsilon\Delta t \left(\frac{1}{Vol} \sum_{all\ faces} \left(A^{+} + A_{v} \right)^{(q-1)} S - R^{(q-1)}\left(k,\omega\right) \right) \right] \begin{pmatrix} \Delta k \\ \Delta\omega \end{pmatrix}^{p} = \begin{pmatrix} \Delta k \\ \Delta\omega \end{pmatrix}^{q} - \varepsilon\Delta t \left(\frac{1}{Vol} \sum_{all\ faces} \left(A^{-} - A_{v} \right)^{(q-1)} S \right) \begin{pmatrix} \Delta k \\ \Delta\omega \end{pmatrix}_{NB}^{p-1}$$

This system is solved with the Gauss-Seidel method using a small number of iterations m. Only the decaying terms of the source term are used in the source flux Jacobian:

$$R^{(q-1)}\left(k,\omega\right) = -\begin{pmatrix} \beta^{*}\omega & 0 \\ \beta^{*}k & 2\beta\omega \end{pmatrix}$$

A second application is time dependent flows using a dual time step algorithm. The time derivative is approximated at each physical time step by a backward difference formula. The resultant system is solved by a modification of the steady state algorithm that treats the lower order terms implicitly. For this to be effective each "steady state" solution must be solved extremely fast. With the original algorithm it typically required 50 subiterations to reduce the residual by 3 orders of magnitude and over 100 iterations to reduce it by 4 orders. With the new preconditioner one typically requires 15 subiterations to reduce the residual by 4 orders resulting in a vastly more efficient algorithm.

2 Results

Figure 1a shows the convergence history for the flow around a RAE2822 wing for several CFL using the implicit RK method in both the fluid and turbulent equations. We used a sequencing of two coarse grids and three levels multigrid scheme on the finest grid and a variety of time steps for both the fluid and turbulent media until machine accuracy is achieved. A second order upwind scheme with a Sweby limiter was used. In figure 1b we compare the new method with a fluid CFL of 100,000 and a turbulent CFL of 20,000 against the original method using CFL=6.5 and an ADI solver for the turbulence equations demonstrating the clear superiority of the new approach.

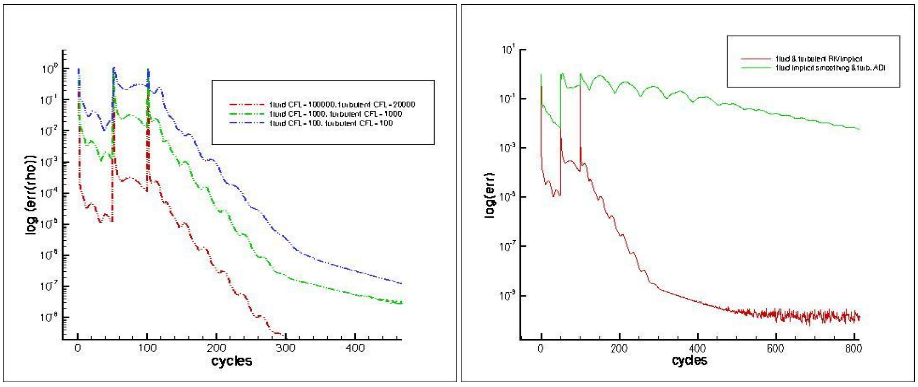

Figure 1: Convergence history of density for RAE2822 airfoil
(a) several CFL for new method (b) new versus original

We next present in Figure 2 results for dual time stepping for flow about a NACA0012 at an angle of attack of 30° with a Baldwin-Lomax turbulence model. Three typical subiteration cycles are shown. The improvement of the new scheme is dramatic.

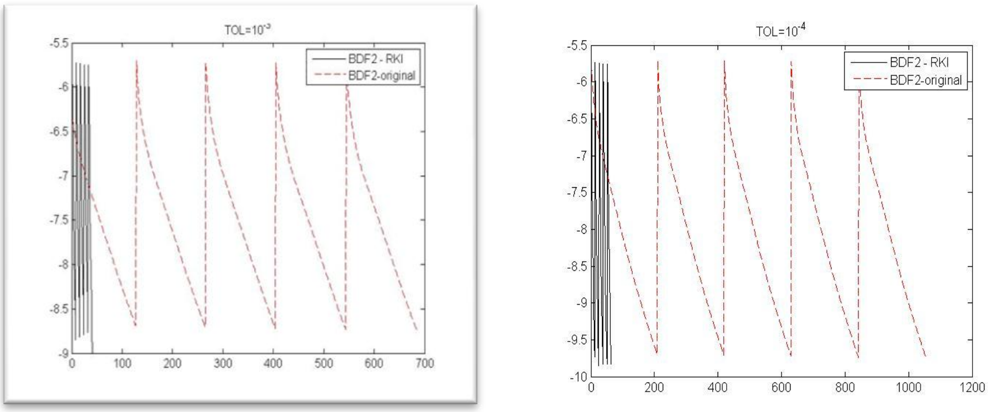

Figure 2: Convergence history dual time step: Comparison of original and preconditioned schemes

Conclusion and Future Work

We solved the Navier-Stokes equations including multi-equation turbulence models and quasi time-dependent problems within a few multigrid cycles. Future work will extend this to combustion.

References

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