Modifications and Clarifications for the Implementation of the Spalart-Allmaras Turbulence Model

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Abstract: We present modifications to the Spalart-Allmaras (S–A) turbulence model targeted toward situations of under-resolved grids and unphysical transient states. These modifications are formulated to be passive to the original model in well resolved flowfields and should produce negligible differences in most cases. We also comment on the appropriate form of S–A for compressible flows and comment on the inclusion of the laminar suppression term for fully turbulent flows. We also present a new analytic solution to S–A for law of the wall velocity.

Keywords: Computational Fluid Dynamics, Turbulence Modeling.

1 Introduction

The Spalart-Allmaras (S–A) turbulence model [1, 2] has been widely used and has proven to be numerically well behaved in most cases. There are, however, situations of under-resolved grids and unphysical transients where discretization of the model can lead to undesired results. Undershoots at the edge of boundary layers and wakes is one such situation. Another is when the modified vorticity $\widetilde{S}$ becomes negative. We propose modifications to the S–A model to remedy these situations. The first is a continuation of S–A for negative $\tilde{\nu}$ solution values. The second is a change in the definition of $\widetilde{S}$ that avoids negative values.

Many applications of S–A target fully turbulent flows, where the flow is essentially turbulent everywhere vorticity is present. For these flows, inclusion of the laminar suppression term (f_{t2}) is effectively optional since it has negligible effect on the resulting flow. This is the case as long the freestream level of $\tilde{\nu}$ is high enough. We comment on the appropriate values of freestream $\tilde{\nu}$ in the presence of the f_{t2} term.

The original S–A references formulated a single partial differential equation (PDE) applicable to both incompressible and compressible flows. Unfortunately, there is confusion in the literature on the compressible form of S–A. We clarify the standard form of S–A for compressible flows and comment on associated jump conditions.

S–A is formulated to admit simple solutions for $\tilde{\nu}$ and the modified vorticity $\widetilde{S}$ for the law of the wall. We present a new analytic solution for the velocity that satisfies S–A in the law of the wall.

2 Discussion

2.1 Negative S-A model

S–A was originally formulated for positive $\tilde{\nu}$ solutions. We formulate a continuation of S–A into the realm of negative solutions with the following properties:

- functions in the PDE are C^1 continuous with respect to $\tilde{\nu}$ at $\tilde{\nu} = 0$
- negative $\tilde{\nu}$ produces zero eddy viscosity
- negative S–A is energy stable with local minimums forced towards positive
- analytic solution is non-negative given non-negative boundary conditions

The original (positive) S–A model admits only non-negative solutions given non-negative boundary and initial conditions. This situation is not always possible discretely, and there are situations on coarse grids and transient states where the turbulence solution may become negative. This is often encountered at the edge of boundary layers and wakes where the turbulence solution is characterized by ramp solutions that transition to constant outer/freestream solutions over a short $O(1/\mathrm{Re})$ region. The rapid transition from large inner to relatively small outer solutions can result in undershoots for discrete solutions. These undershoots may cross zero, requiring some action to continue. The common practice in these situations has been to clip updates eliminating negative solution values. However, clipping updates prevents the convergence of discrete PDE residuals and hampers efforts to quantify discrete truncation and solution errors. Another approach to eliminate negative solutions is to formulate positive discrete operators. This was done in Ref. [1] but the resulting discretized advection operator was only first order accurate. Nguyen, Persson and Peraire [3] modify the discretization with artificial dissipation to prevent undershoots in the region of the boundary layer edge. Though promising, the method cannot be shown to be positive and may not prevent negative undershoots in all situations.

We formulate a negative version of S–A to deal with situations of negative undershoots. Although an analytic continuation of S–A, its primary purpose is to address issues with under-resolved grids and non-physical transient states in discrete settings. The present formulation was provided to Oliver and appears in his PhD thesis [4]. Unfortunately, Oliver's presentation contains a minor discrepancy concerning the f_{t2} term. We include the term; he did not. Including f_{t2} in positive S–A aids in producing a negative continuation that is both C^1 continuous and provides a production term that is always positive for negative $\tilde{\nu}$ — a property that ensures energy stability and prevents negative values in the analytic limit.

2.2 Preventing negative values of modified vorticity $\widetilde{S}$

In physically relevant situations, the modified vorticity $\widetilde{S}$ should always be positive with a value that never falls below $0.3S$, where S is the vorticity magnitude. However, discretely this is not always the case. It is possible for $\widetilde{S}$ to become zero or negative due to the fact that f_{v2} is itself negative over a range of χ. Negative $\widetilde{S}$ in turn disrupts other S–A correlation functions. We present a modified form of $\widetilde{S}$ that is identical to the original for $\widetilde{S} > 0.3S$, but remains positive for all nonzero S and is C^1 continuous.

2.3 Inclusion of f_{t2} and appropriate freestream values of $\tilde{\nu}$

The f_{t2} laminar suppression term was included in the model to prevent spurious growth of small $\tilde{\nu}$ in the presence of vorticity. Its primary purpose is to prevent premature transition in laminar boundary layers upstream of trips (the f_{t1} term). For fully turbulent flows with adequately large freestream $\tilde{\nu}$ values (e.g. $\tilde{\nu} = 5\nu$), inclusion of the f_{t2} term produces negligible effects. However, if freestream values are chosen near or below the basin of attraction for f_{t2}, then solutions could possibly revert to laminar flow. We further comment on appropriate values of freestream $\tilde{\nu}$ for fully turbulent flows. We also reiterate the findings of Rumsey and Spalart [5] in choosing freestream conditions.

2.4 Compressible form of S-A

Confusion exists in the literature over the formulation of S–A for compressible flows. We reaffirm that the formulation presented in Ref. [2] is applicable to both incompressible and compressible flows, and it should be considered the standard form for compressible. An equivalent conservation form can be constructed by combining S–A with the mass conservation equation. We also comment on jump conditions across shocks.

2.5 Analytic Solution for Law of the Wall Velocity

S–A is formulated to permit the following simple solution for law of the wall,

$$\tilde{\nu} = \kappa u_\tau d, \qquad \widetilde{S} = \frac{u_\tau}{\kappa d},$$

where u_τ is the shear stress velocity, d is distance from the wall and $\kappa = 0.41$ is von Karman's constant. Integrating this solution to give an explicit expression for velocity has proven difficult due to the complexity of the eddy viscosity correlation function f_{v1}, which also enters into the definition of $\widetilde{S}$. We have now succeeded and present a new analytic solution for velocity $u(d)$ that is consistent with S–A in the law of the wall region.

3 Full Paper

The full paper will include expanded presentation of the preceding five topics, including specific equations and example cases.

References

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