An efficient Newton-Krylov-Schur parallel solution algorithm for the steady and unsteady Navier-Stokes equations

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Abstract: We present a novel and efficient parallel Newton-Krylov-Schur algorithm for the solution of the Navier-Stokes equations. The governing equations are discretized using summation-by-parts operators of various orders, with boundary condition imposition and interface coupling achieved with the use of simultaneous approximation terms. For unsteady flows, the solution is integrated in time with explicit first stage, singly diagonally implicit Runge-Kutta methods of various orders. The discretized system of equations is solved through an inexact-Newton method with an approximate-Schur parallel preconditioner. The parallel capabilities of the algorithm can be leveraged to efficiently obtain steady solutions of complex turbulent flows, as well as to simulate unsteady transitional and turbulent flows based on implicit large-eddy and direct simulations.

Keywords: Newton-Krylov, Approximate-Schur Preconditioner, ESDIRK, Steady Flows, Unsteady Flows, Parallel Computations, SBP-SAT Discretization.

Newton-Krylov algorithms [1] have been shown to be efficient in serial [2, 3] and parallel implementations, the latter using the additive-Schwarz [4, 5] or approximate-Schur [6, 7] preconditioners. With the use of implicit time integration, such as explicit first stage, singly diagonally implicit Runge-Kutta (ESDIRK) methods, the algorithm has been applied to unsteady laminar and turbulent flows [8].

In this paper we present an extension of the efficient Newton-Krylov-Schur parallel implicit algorithm for the Euler equations of Hicken and Zingg [7] to the (Reynolds-averaged) Navier-Stokes equations. The algorithm uses summation-by-parts operators and simultaneous approximation terms (SATs) to discretize the governing equations in space, readily lending itself to a parallel implicit structured multi-block implementation. The Spalart-Allmaras one-equation turbulence model [9] is used to simulate turbulent effects. SATs weakly enforce boundary conditions and inter-block continuity through a penalty method. To obtain steady solutions, a globalized inexact-Newton method is used. The ESDIRK method is used to evolve unsteady solutions in a time accurate manner. For both steady and unsteady problems, at each nonlinear iteration the resulting large sparse system of linear equations is solved by the Krylov iterative solver flexible GMRES, using the approximate-Schur preconditioner.

Figure 1 presents C_p distributions at two spanwise sections for a fully turbulent steady transonic flow over the ONERA M6 wing, along with the convergence history. This case was computed using 128 processors on a 15.1 million node grid, obtaining a steady state solution by reducing the residual by 12 orders of magnitude in 89 minutes.

Figure 2 presents the instantaneous Q-criterion (Q=1) colored by streamwise vorticity for transitional flow over an extruded SD7003 wing, as well as a C_p plot, averaged over $\Delta t = 8$, overlaid by results from Zhou and Wang [11], and Galbraith and Visbal [12]. The implicit large-eddy simulation (ILES) was computed with 448 processors on a 7.87 million node grid using ESDIRK4, averaging about 197 time steps per hour with $\Delta t = 10^{-3}$.

The final paper will present the details of the full Newton-Krylov-Schur algorithm, along with

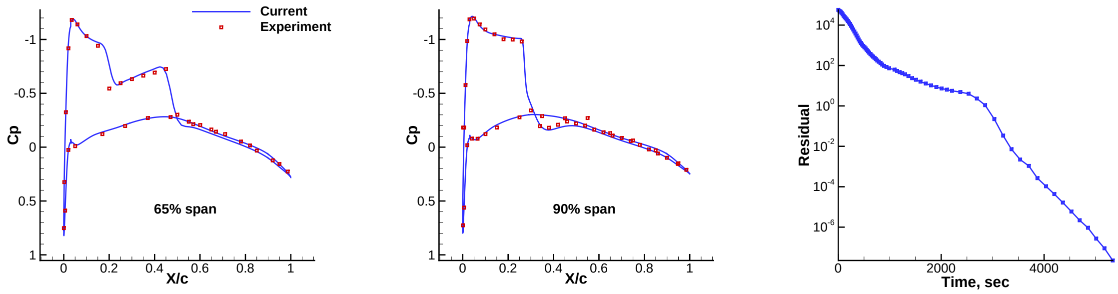

Figure 1: C_p distributions compared to experimental results of Schmitt and Charpin [10], and convergence plot for steady transonic flow (Ma=0.8395, Re=11.72e6, α=3.06°)

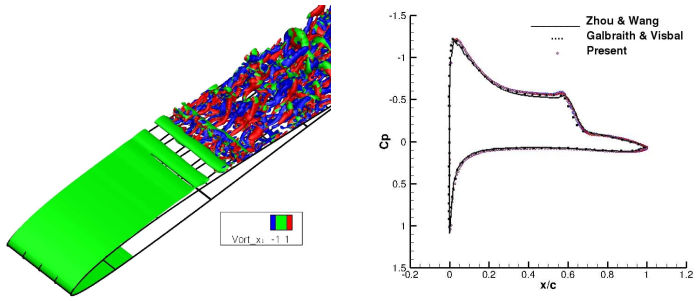

Figure 2: Q-criterion (Q=1) coloured by streamwise vorticity and C_p plot overlayed by results from Zhou and Wang [11], and Galbraith and Visbal [12] for the SD7003 wing (Ma=0.2, Re=60e3, α=4°)

extension to higher order. A variety of steady and unsteady flow problems will be investigated, and grid convergence studies will demonstrate the efficiency of the algorithm, as well as the relative efficiency of various orders of accuracy.

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