A Semi-Unstructured Multi-Block Fourth-Order Energy-Stable Weighted Essentially Non-Oscillatory Finite Difference Scheme^{*}

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Abstract: A new methodology is devised and demonstrated for accurately capturing discontinuities in multi-block finite difference simulations of hyperbolic partial differential equations. The fourth-order energy-stable weighted essentially non-oscillatory (ESWENO) finite difference scheme on closed domains is combined with simultaneous approximation term (SAT) weak interface and boundary conditions. Smoothness of grid-spacing across subdomain interfaces is not required. WENO stencil-biasing is truncated near subdomain boundaries and only collocated interface points are communicated between neighboring subdomains. The methodology is demonstrated for significant jumps in grid-spacing across subdomain interfaces. Results are presented for the linear scalar hyperbolic wave equation in one and two dimensions and the Euler equations in one and two dimensions. It is demonstrated that this methodology allows strong discontinuities to be passed across subdomain interfaces without significant distortion. Moreover it is demonstrated that the methodology provides stable and accurate results even when large differences in the grid-spacing exist, whereas strong imposition of the interface conditions causes noticeable oscillations. Weak subdomain interdependence, low subdomain-to-subdomain message-passing overhead, and ease of local grid refinement make the new methodology promising for scalable massively-parallel simulations and complex geometries.

Keywords: Multi-Block Finite-Difference Schemes, Boundary Conditions, High-Order Numerical Methods, Shock-capturing, Computational Fluid Dynamics.

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