Spectral accuracy of reconstruction on arbitrary unstructured grids

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Abstract: We study high-order (up to $O(\Delta\ell^{12})$) least-squares reconstruction, both linear and WENO, on arbitrary unstructured grids. The order-of-accuracy of the schemes is evaluated on different types of unstructured grids by calculating the error of the solution of the advection equation with different orientations of the advection-velocity-vector. The strategy for building the reconstruction stencils and the WENO substencils is discussed in detail. The spectral accuracy of the reconstruction procedure is studied analytically for several regular unstructured grids. Finally, the WENO schemes are evaluated for several testcases for the Euler equations of gasdynamics.

Keywords: High-Order Schemes, Unstructured Meshes, Multidimensional Least-Squares Reconstruction, WENO Reconstruction, Advection Equation, Euler Equations.

1 Introduction

The relative advantages of cell-centered and vertex-centered finite-volume (FVs) schemes [5] are best understood, especially for high-order schemes, as the comparison of the relative performance of high-degree polynomial least-squares approximations [6] on different control volumes, relative to Delauney triangulations [2] or Voronoi tesselations [2] or some other connectivity between points [1]. We study in the present paper the performance of very-high-order least-squares reconstruction on different types of general polygonal grids.

2 Present Approach

As an example, consider linear (in the sense of Godunov's theorem [7]) least-squares schemes [6] for the 2-D advection equation on a square domain (Fig. 1). The points of a regular Cartesian grid can be triangulated in different ways (Fig. 1), and the dual-grids composed by the control-volumes constructed by joining the barycenters of the cells around each vertex are general polygonal grids (Fig. 1), while the dual of a basic quasi-Delauney triangulation with equilateral triangles is a quasi-Voronoi tesselation of the square (Fig. 1). The solution of the 2-D advection equation on progressively refined grids of each type, using polynomial least-squares reconstruction of different orders (polynomial degree), with various strategies for stencil-construction (face-neighbours or vertex-neighbours) and least-squares weighting (distance-weighted or unweighted), and for different orientations of the advection-velocity-vector, is compared with the exact solution (Fig. 1). The results are then correlated with different widely used measures of grid-quality. We then study nonlinear WENO schemes, up to $O(\Delta\ell^{12})$, for the same problems. We study in particular systematic strategies for the construction of the substencils.

For regular unstructured grids (Fig. 1) it is possible to obtain analytical expressions of the reconstructing polynomial coefficients, and compute exactly, using symbolic calculation, the fluxes at the Gauss integration points on the cell-faces. We have used this approach to study the spectral accuracy of least-squares-reconstruction discretizations of the advection equation, and compare with the results obtained by the aforementioned numerical experimentation.

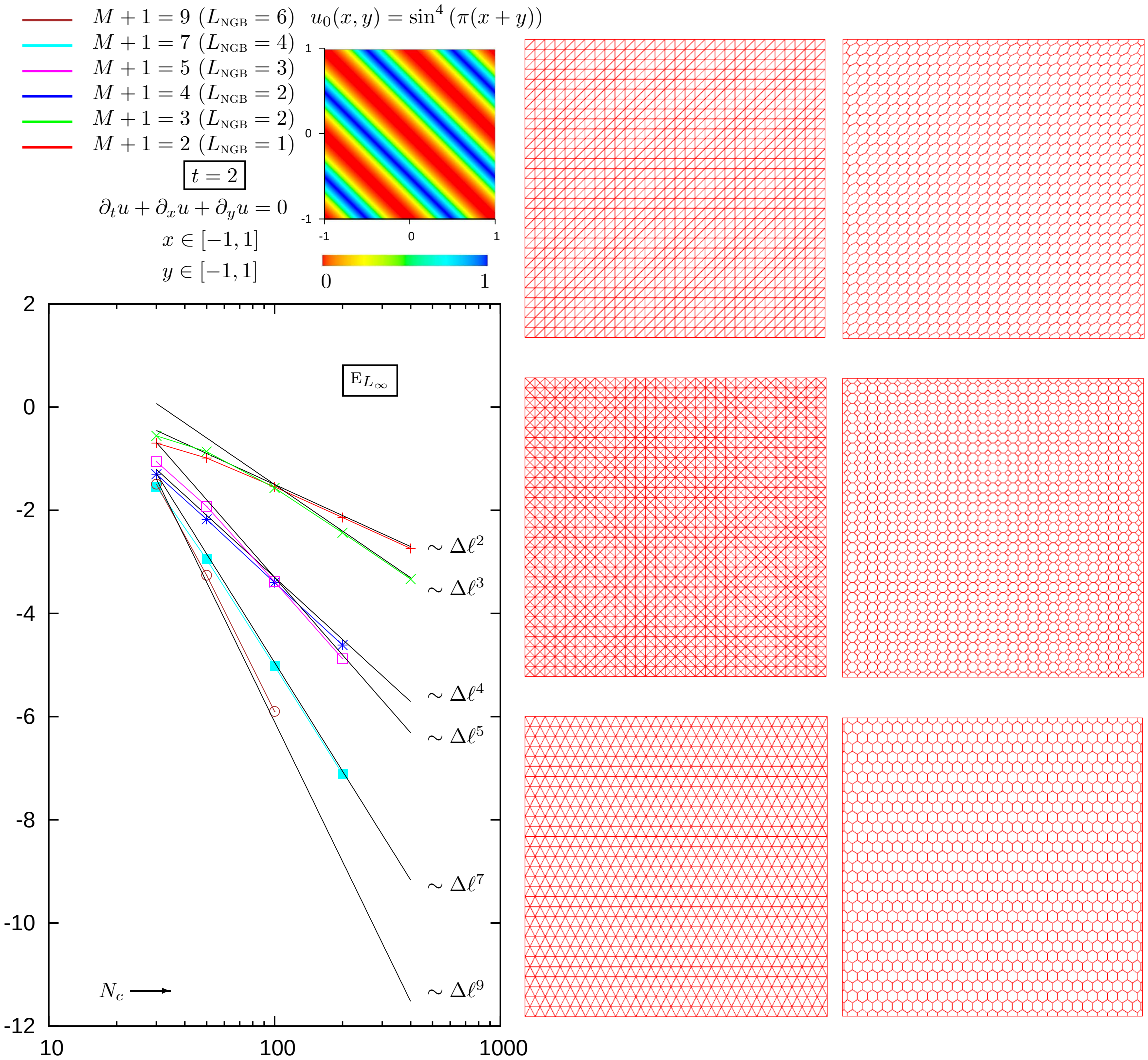

Figure 1: Examples of different triangular and general polygonal grids (dual to the triagular) and order-of-accuracy studies for the 2-D advection equation (with linear SSPRK time-integration of the corresponding order [4]).

Finally, this procedure is extended to the Euler equations of gasdynamics, using characteristic variables reconstruction and an exact Riemann solver [3], and applied to standard test-problems.

3 Conclusion and Future Work

In the present work we studied the order-of-accuracy and the spectral accuracy of linear and WENO schemes on general unstructured meshes, and tested very-high-order unstructured WENO schemes for the Euler equations, lowering the order-of-accuracy at nonperiodic boundaries. Ongoing work concentrates on very-high-order-accurate WENO schemes on general unstructured meshes, using biased high-order stencils at the boundaries.

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