# Discretization Methods for Extremely Anisotropic Diffusion

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**Abstract:** Special discretization methods are presented for multi-dimensional diffusion operators with strongly anisotropic diffusion. These operators occur in the magnetohydrodynamic (MHD) equations describing tokamak plasma physics.

*Keywords:* Tokamak plasmas, magnetohydrodynamics, strongly anisotropic diffusion, special discretization methods.

## 1 Introduction

In tokamak fusion plasmas there is extreme anisotropy in heat conduction coefficients due to the high temperature and large magnetic field strength. This allows diffusive processes, to effectively be aligned with magnetic field lines. Heat conduction in tokamak fusion plasmas can be up to $10^{12}$ times larger in magnetic-field-aligned direction than in the direction normal to that. This anisotropy puts severe requirements on numerical methods for MHD; small misalignment of the grid may cause the perpendicular diffusion to be significantly polluted by the numerical error in approximating the parallel diffusion. A common remedy is to apply coordinates aligned with the magnetic field. However, this approach runs into problems in case of crossing field lines, e.g., at $x$-points and where there is magnetic reconnection. It is useful therefore to consider discretization methods which are more tolerant to the misalignment of the grid with the magnetic field lines, ultimately to allow for the use of cartesian, non-aligned grids.

Besides spurious diffusion in perpendicular direction, two other problems that might arise due to strongly anisotropic diffusion are: *(i)* non-positivity near high gradients and *(ii)* stagnation or loss of convergence [1].

To enable accurate numerical simulation of phenomena which rely heavily on the resolution of the perpendicular temperature gradient, here we present discretization methods that are accurate in case of strongly anisotropic diffusion and misalignment.

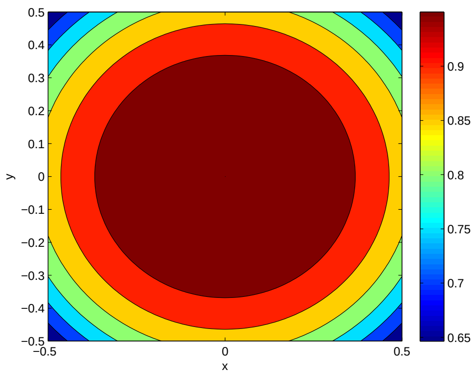
## 2 Problem Statement

Our model problem is the unsteady heat equation

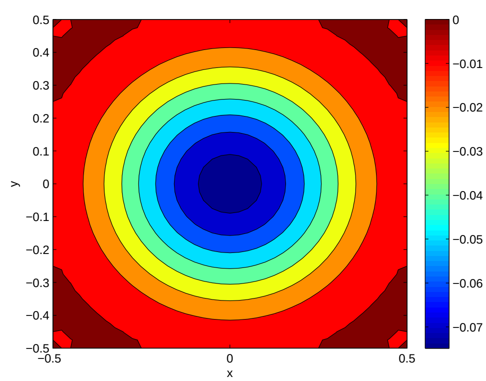
$$\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} + f, \quad \mathbf{q} = \mathbf{D}\nabla T, \quad \mathbf{D} = \begin{pmatrix} D_\parallel b_1^2 + D_\perp b_2^2 & (D_\parallel - D_\perp) b_1 b_2 \\ (D_\parallel - D_\perp) b_1 b_2 & D_\perp b_1^2 + D_\parallel b_2^2 \end{pmatrix}, \tag{1}$$

where $T$ represents temperature, $b_1, b_2$ the components of the unit direction vector of the magnetic field line with respect to the coordinate axes, $f$ some source term, and $D_\parallel$ and $D_\perp$ the parallel and perpendicular diffusion coefficients.

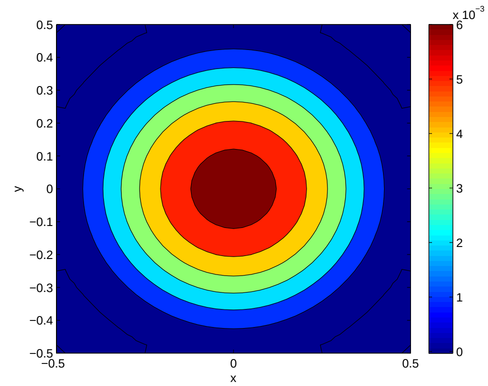
For a 2D test case with $\frac{D_\parallel}{D_\perp} = 10^6$, in Figure 1 we give the exact discrete solution on a $40 \times 40$ grid (Figure 1a) and the distributions of the solution error corresponding with a conventional discretization method (Figure 1b) and a novel method that we developed (Figure 1c).




(a) Exact discrete solution




(b) Solution error, conventional method




(c) Solution error, novel method

Figure 1: Exact discrete temperature distribution and solution errors for closed field line case

## 3 Conclusion and Future Work

The novel method is significantly more accurate than the conventional method, but it is susceptible for overshoots and non-positivity. It still needs to be improved for monotonicity and positivity.

## References


[1] S. Günter, Q. Yu, J. Krüger and K. Lackner. Modelling of heat transport in magnetised plasmas under non-aligned coordinates. *J. Comput. Phys.*, 209:354-370, 2005.