Local Stability Enhancement of Immersed Boundary/Interface Methods.

Christoph Brehm*, and Hermann F. Fasel**
Corresponding author: christoph.brehm@nasa.gov or cbrehm@email.arizona.edu

* Science and Technology Corporation, Moffett Field, CA, USA.
** University of Arizona, AZ, USA.

Abstract: The objective of this work is to present a novel, robust, high-order accurate Immersed Interface/Boundary Method (*IIM*). In contrast to other immersed methods, which lack a posteriori check of the numerical stability, the method presented herein combines a local Taylor-series expansion at irregular grid points with a local stability condition as part of the design process. Furthermore, this work demonstrates that the local stability constraint is sufficient to ensure a globally stable method as long as the DFL-number is less than its limiting value. To validate our novel approach, several test-cases from fluid mechanics and electromagnetics are presented.

Keywords: Immersed Boundary, Numerical Stability Analysis.

1 Introduction

For simulations of highly complex geometries frequently encountered in many fields of science and engineering, the process of generating a high-quality, body-fitted grid is complicated and time-intensive. Thus, one of the principal goals of contemporary Computational Fluid Dynamics (*CFD*) is the development of numerical algorithms that can deliver computationally efficient and highly accurate solutions for a wide range of applications involving multi-physics problems, e.g. Fluid Structure Interaction (*FSI*). In designing such methods, Immersed boundary/interface methods provide considerable advantages over conventional approaches, especially for flow problems containing moving boundaries. Immersed Boundary (*IB*) methods have been around for several decades and have appeared in various forms since they were first introduced in the 1970s. These methods have become increasingly relevant and are considered valuable beyond their application as a non-traditional approach for numerically solving initial boundary-value problems on domains with complex geometries. One considerable advantage of these methods is their simplicity in generating grids, independent of the complexity of the geometry. The grid generation process for body-conformal structured or unstructured grids is generally cumbersome, because the grid generation process aims at generating a grid that offers sufficient local resolution without exceeding a maximum number of total grid points. For anything but the simplest geometries, these conflicting requirements can lead to deterioration in grid quality, thereby negatively impacting the accuracy and convergence properties of the solver (see Ferziger and Peric [1]). For simulating flows with moving or deforming boundaries, the use of body-conformal grids requires generating a new grid at each time-step in addition to a procedure capable of projecting the solution onto this new grid (Tezduyar [2]). These two characteristics associated with simulating flows with moving boundaries may negatively impact the accuracy, robustness, and computational cost of the numerical solution method. Particularly, in cases where the boundary exhibits large motions, the current body-conformal grid approaches complicate the solution procedure. As a result, Immersed Interface Methods (*IIMs*) (or immersed boundary methods) are a promising alternative for body-fitted grid approaches. For flows containing moving boundaries, immersed interface/boundary methods provide clear advantages over conventional approaches (e.g. Arbitrary Lagrangian-Eulerian (*ALE*) approach). They provide a convenient numerical technique that allows the body to move or deform on a stationary non-deforming Cartesian grid. While these methods simplify the grid generation process, a detailed mathematical understanding of the immersed interface method is necessary to avoid a negative impact of the boundary treatment on the convergence behavior and the accuracy of the numerical scheme.

1.1 Immersed Boundary Method

This paper introduces a novel approach for designing immersed boundary/interface methods for solving advection-diffusion type equations. In addition to conventional immersed schemes, which usually only consider the accuracy (local truncation error) at irregular grid points, this paper demonstrates that it is possible to consider a local stability condition for the derivation of the boundary stencils. This novel concept provides improved robustness to the numerical scheme without compromising accuracy. One of the key points emphasized in this paper is that when solving a coupled partial differential equation (in space and time), the coupling between the temporal and spatial discrete operators need to be considered in order to derive a numerical scheme that not only achieves the desired order of accuracy, but also fulfills the stability requirements. Two different local stability constraints are formulated herein: the first is based on a necessary condition, and the second is based on a sufficient condition for the stability of the numerical scheme. The local stability constraints enforce an additional relationship between the coefficients of a boundary stencil in order to achieve a stable time-explicit immersed scheme. In deriving this special immersed treatment, the localization assumption must be valid in order to ensure that the boundary stencil can be isolated from the remainder of the computational grid. Indeed, this paper demonstrates, both numerically and analytically, that the localization assumption is valid as long as the DFL-number does not reach a limiting value. Furthermore, the stability analysis results and the numerical validation cases presented in this work provide strong evidence that superior stability characteristics can be achieved with the novel immersed scheme. Further several applications of the immersed boundary/interface method are presented to show the current approach's versatility. It must be re-emphasized that this paper explores a novel design concept for immersed boundary/interface methods by utilizing local stability constraints, which are based on very thorough derivations and numerical analysis. Moreover, the idea of locally adjusting the grid stencil in order to closely mimic the spectral characteristics of an "ideal scheme" are applicable to other numerical applications, such as locally enforcing the maximum principle for elliptical equations, preserving continuity or an energy norm for interpolation operators, and others.

2 Results and Summary

After discussing the theoretical concept of the novel immersed boundary/interface method for different types of equations, simulation results where the current approach was utilized are presented. The method presented herein has been used in an incompressible Navier-Stokes solver to investigate the stability characteristics of a Blasius boundary layer flow over a wavy wall. This test-case is very challenging since for stability calculations the accuracy in the vicinity of wall is crucial for the precise prediction of the stability characteristics of the mean flow. Different parameters such as the shape, height, and the spacing of the wall waviness were investigated in this study. In another application, the novel immersed boundary/interface method was used together with an incompressible Navier-Stokes solver to study the flow through constricted tubes, which serve as a model for stenosed arteries (see Brehm *et al.* [3]). The immersed boundary approach is very attractive for the simulation of biofluid dynamics problems. The artery tree in the human body provides a great variety of different shapes with complicated three-dimensional bends and continuously changing artery diameters. The novel immersed boundary approach has also been used for Temporal Direct Numerical Simulations (*TDNS*) of instability waves in a compressible Mach 6 boundary with porous walls. The main parameters of the porous wall are the depth of the porous layer, d, the porosity, ϕ, and the number of pores per wave length, n_p. The porosity is defined as the ratio of the porous surface to the total surface. A parameter study revealed the geometric dimensions for which the porous coating most effectively attenuates the growth of disturbances in the boundary layers (see Hader and Fasel [4]). For the optimal parameter combination, significant stabilization of the temporal/spatial growth of the instability waves is observed in experiments and computations.

References

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