Simulation of Decaying Two-Dimensional Turbulence Using Kinetically Reduced Local Navier-Stokes Equations

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Abstract: Kinetically reduced local Navier-Stokes (KRLNS) equations are applied for simulation of decaying two-dimensional (2-D) homogeneous isotropic turbulence in order to demonstrate it's capability to capture the correct transient behavior. The numerical results obtained by the KRLNS equations are compared with those obtained by the artificial compressibility method (ACM), the lattice Boltzmann method (LBM) and the pseudo-spectral method (PCM). The divergence as a function of time in the KRLNS method is compared with that of the ACM. It is confirmed that the KRLNS method can capture the correct transient behavior without use of sub-iterations due to a smoothing effect.

Keywords: Unsteady Incompressible Viscous Flow, Kinetically Reduced Local N-S Equations, Artificial compressibility Method, Lattice Boltzmann Method.

1 Introduction

Recently, an alternative thermodynamic description of incompressible fluid flows was suggested in the form of kinetically reduced local Navier-Stokes (KRLNS) equations and the capture of the correct time dynamics was studied [1]. In this paper, in order to investigate the capability to capture the correct transient behavior of the KRLNS equations, numerical simulations of decaying two-dimensional turbulence are carried out and the results are compared with the solutions obtained by the artificial compressibility method (ACM), the lattice Boltzmann method (LBM) and the pseudo-spectral method (PSM).

2 Kinetically Reduced Local Navier-Stokes Equations and Numerical Method

The classical incompressible Navier-Stokes equations consist of the equation for the velocity

$$\partial_t u_\alpha + u_\beta \partial_\beta u_\alpha + \partial_\alpha p = \frac{1}{\mathrm{Re}} \partial_\beta \partial_\beta u_\alpha \text{, (1)}$$

and the incompressibility constraint

$$\partial_\alpha u_\alpha = 0 \text{, (2)}$$

where $\mathbf{u}$ is the fluid velocity, p is the pressure and Re is the Reynolds number. In the ACM, the time derivative of the pressure is introduced into the continuity equation for coupling between the pressure and the velocity. The continuity equation (2) is then written as

$$\frac{1}{\delta}\partial_t p = -\partial_\alpha u_\alpha \text{, (3)}$$

where δ is the artificial compressibility parameter. t is an auxiliary variable that can be related to the physical time. To satisfy the continuity equation (2), the sub-iterations at each time step is mandatory.

In the form of KRLNS equations, the pressure equation (3) is replaced by

$$\partial_t G = -\frac{1}{Ma^2}\partial_\alpha u_\alpha + \frac{1}{\mathrm{Re}}\partial_\beta \partial_\beta G \text{ ,} \quad p = G + \frac{u^2}{2} \text{ (4)}$$

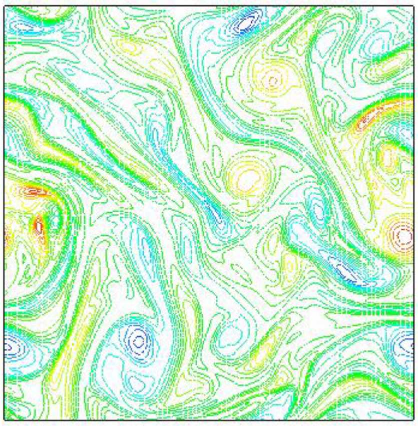
,where Ma is the Mach number, G is the grand potential. Retaining the term $1/\mathrm{Re}\,\partial_\beta\partial_\beta G$ is crucial for capturing the correct transient behavior without sub-iterations. In the numerical method for solving the KRLNS equations, a central difference scheme is used for the spatial discretization and 4 stage Runge-Kutta method is used in the time integration.

3 Numerical Results

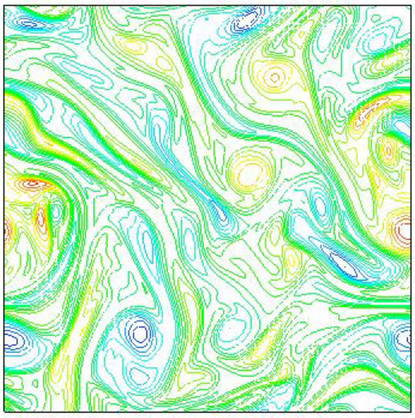
The vorticity contour plot obtained by the KRLNS equations for decaying 2-D turbulence at Re = 10000 on a uniform 256×256 Cartesian grid is shown in Figure 1(a), while Figure 1(b) shows that obtained by the LBM. The plot obtained by the PSM, which is a standard approach to the direct simulation of turbulence, is shown in Figure 1(c). The agreement among the three simulations is excellent. The divergence as a function of time for the KRLNS equations is shown in Figure 2. The adjustable parameters for the KRLNS equations is Ma = 0.02, $\Delta t = 1\times10^{-4}$. It is confirmed that the KRLNS approach can keep the divergence fluctuation at the 10^{-2} level, while that in the ACM is much larger.

References

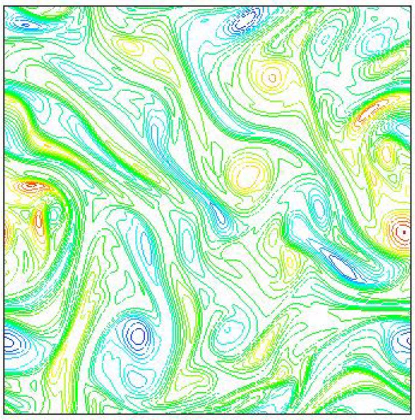
[1] S. Borok, S. Ansumaliand I. V. Karlin,Kinetically reduced local Navier-Stokes equations for simulation of incompressible viscous flows, *Physical review E 76*, 066704, (2007).



(a) KRLNS



(c) PSM



(b) LBM

Figure1 Comparison of vorticity contour plots at time t=1.

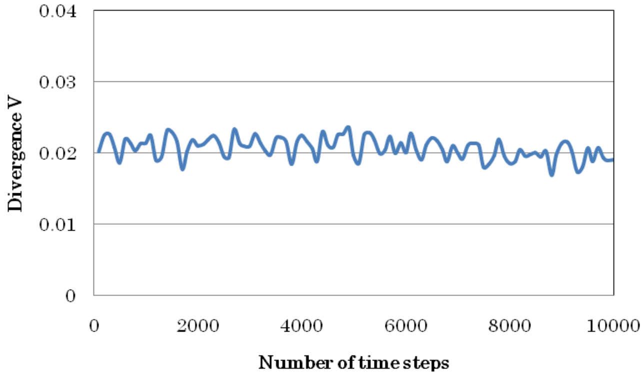


Figure2 Divergence histories obtained by the KRLNS.