Low Dispersion Finite Volume Scheme Based on MDCD Reconstruction

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Abstract: The minimized dispersion and controllable dissipation (MDCD) scheme has shown desirable properties of both dispersion and dissipation. The scheme originally designed in the finite difference frame is capable to handle flow discontinuities and resolve a broad range of length scales. In this paper, low dispersion finite volume formulations based on MDCD reconstruction are presented to better handle rich flow features and complex geometry encountered in many practical engineering applications. The resolution and robustness of the proposed scheme are demonstrated by its application to several benchmark and engineering test cases.

Keywords: Low Dispersion, Controllable Dissipation, Finite Volume Scheme, Reconstruction.

1 Introduction

The simulations for turbulence with all possible length scales, computational aero-acoustics (CAA) and other flow field with rich flow features require low dispersion and low dissipation schemes.It is generally accepted that the optimal dispersion should be minimal according to some chosen criteria while the optimal dissipation is often problem dependent. So Sun et al. [1] presented the MDCD scheme, minimizing dispersion error and leaving one free parameter for adjusting dissipation. The key feature of this scheme is that the dissipation and dispersion are independent and therefore can be controlled separately. Meanwhile a hybrid scheme, combining the MDCD scheme and WENO scheme, is constructed to compute flow with discontinuities.

2 Problem Statement

The optimized schemes above are originally designed based on the finite difference approach. However, a finite volume formulation is superior to a finite difference approach because of its easier framework to handle the irregular geometry and moving boundaries. It can be advantageous to adopt the finite volume approach, which ensures fluxes estimated from different sides of the same surface are identical. Such a requirement is particularly important when nonlinearity is involved, as is typically the case in shock and turbulence aspects of the aero-acoustic computations. In the construction of the finite volume MDCD scheme, we must handle the cell average instead of the point value of the conservation variables. Following the spirit of the MDCD, the

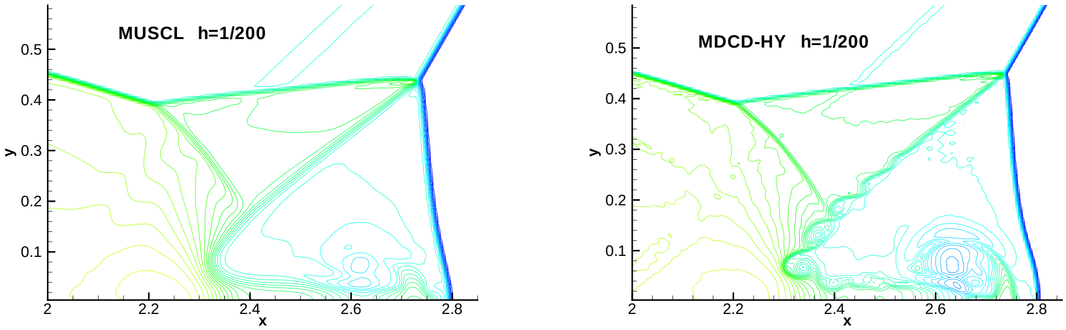

Figure 1: Density contours for double Mach reflection problem; MUSCL (left) and MDCD-HY (right).

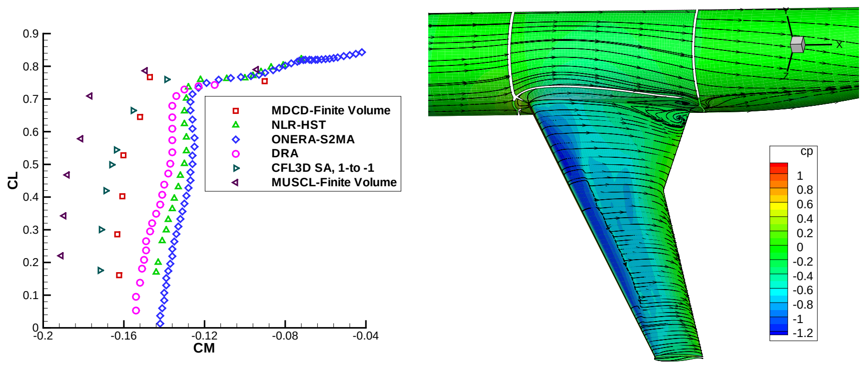

Figure 2: Integration of pitching moment coefficient (left) and streamlines of DLR-F4 (right).

condition is derived under which the dissipation and the dispersion are independent of each other. Using this condition, a high resolution finite volume scheme is designed with minimal dispersion and adjustable dissipation. And furthermore the adjustment of the dissipation will not change the optimized dispersion property of the scheme. Figure 1 shows that the hybrid scheme MDCD-HY gives a better resolution than MUSCL for the contact discontinues and the associated jet. The results of DLR-F4 test are shown in Figure 2 in which the finite volume MDCD scheme shows more accurate results than the CFL3D using finer grids.

3 Conclusion

The MDCD scheme originally designed in the finite difference frame is adapted to reconstruction of the left and right states at each interface necessary to compute the inviscid fluxes. Several benchmark tests are conducted and the low dispersion finite volume schemes show excellent resolution characteristics.

References

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