Design and Verification Methodology of Boundary Conditions for Finite Volume Schemes

A. Katz*, O. Tong*, and V. Sankaran**
Corresponding author: aaron.katz@usu.edu

*Utah State University, USA, **Air Force Research Laboratory, USA.

Abstract: A general comprehensive strategy is presented for the design, verification, and implementation of boundary conditions for finite volume schemes. We incorporate boundary conditions through reconstructed states used in numerical flux formulae at the boundary. A novel aspect of this work is the application of manufactured solutions directly at the boundary equations to verify accuracy.

Keywords: Boundary Conditions, Verification, Manufactured Solutions.

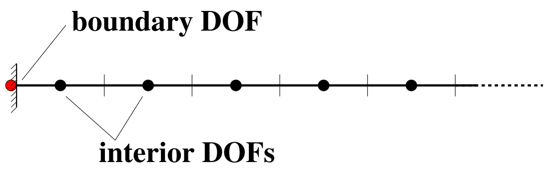
1 Introduction

The[1] objective of this paper is to establish a general design methodology for the implementation and verification of boundary conditions for finite volume (FV) schemes. Unlike finite element approaches [1], a rigorous framework for FV boundary treatments has proven elusive since the boundary conditions do not completely specify the boundary states. As a consequence, various ad hoc methods have appeared in the literature [2]. In this work, three simple criteria are used to define suitable boundary conditions: (1) physical consistency, (2) discrete conservation throughout the entire domain interior, and (3) preservation of the order of accuracy. We describe a new general approach for the implementation of FV boundary conditions through reconstructed states of a numerical flux. Importantly, we present a novel approach for the verification of these boundary conditions using the method of manufactured solutions (MMS). For the final paper, we will present practical demonstrations to validate our approach.

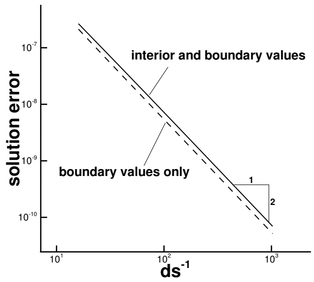
2 Boundary Condition Approach and Verification

The boundary condition strategy is illustrated in Figure 1(a). At interior degrees of freedom we solve the governing PDEs. At domain boundaries, we place new boundary degrees of freedom (BDOF) at the quadrature points of boundary cells (the shaded dot in Figure 1(a)). At these locations we define a new set of equations governing the boundary conditions. These equations involve physically relevant combinations of Dirichlet, Neumann, characteristic, or the equations of motion themselves, and may be expressed as $\Omega(Q) = 0$. For example, at a no slip constant temperature wall, a logical choice is $\Omega(Q) = [\partial p/\partial n \quad u \quad v \quad T - T_{wall}]^T$, where $\boldsymbol{n}$ is the wall normal, and T_{wall} is a specified temperature. By placing the BDOFs at cell quadrature points, we use these values directly as inputs to numerical flux formulae involving reconstructed states, as in $F_{i-1/2} = 1/2[F(Q_R) + F(Q_{BDOF})] - |A|/2(Q_R - Q_{BDOF})$ This automatically enforces upwinding and satisfies discrete conservation for interior cells.

[1](Approved for public release; distribution unlimited) (PA# 111055)



(a) location of boundary degree of freedom



(b) density error using MMS procedure for no-slip wall

Figure 1: Boundary condition implementation and verification.

This method is verified using MMS, which we previously used to determine the accuracy of interior discretizations [4]. Here, we extend the MMS procedure to apply directly at the BDOF locations as well. This is different from the procedure of Choudhary et al. [3] in which MMS solutions are devised which must satisfy given boundary conditions. Our procedure results in a modified boundary equation, $\Omega(Q) = S_b(\boldsymbol{x})$, where S_b accounts for the fact that arbitrary manufactured solutions do not satisfy the boundary conditions generally. The advantage of using MMS on the boundaries is that we can directly assess the accuracy at boundaries as well as the interior scheme without taking great pains to concoct solutions which already satisfy the boundary conditions. An example result of this procedure for the no-slip wall condition above is shown in Figure 1(b), showing second order accuracy.

3 Future Work

For the final paper, detailed descriptions of a variety of boundary conditions will be presented, along with implementation details and verification results. We will include inflow, outflow, fixed mass flux, extrapolation, constant pressure, characteristic, and inviscid/viscous wall treatments. Detailed MMS refinement studies will be performed for these boundary conditions. Additionally, 2D validation cases will be performed to assess physical accuracy, including Ringleb flow, a Blasius boundary layer, and steady inviscid and viscous circular cylinder and airfoils.

References

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