
Oral presentation | Numerical methods

Numerical methods-III

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[7-A-02] Assessment of Immersed Boundary Method Suitable for Kinetic Energy and Entropy Preserving Scheme

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Assessment of Immersed Boundary Method Suitable for Kinetic Energy and Entropy Preserving Scheme

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1 Introduction

Kinetic Energy and Entropy Preserving (KEEP) scheme [1,2] is a recently proposed scheme that enables long-time stable calculations of turbulence without artificial viscosity by conserving kinetic energy and entropy, and it shows effectiveness from analyses on the framework using the Building-Cube-Method (BCM) [3]. To fully exploit the conservative property of the KEEP scheme, the conservation at the wall boundary must be satisfied. The immersed boundary method (IBM) is a wall modeling method, particularly useful for complex geometries. However, IBM does not usually satisfy the conservatism of the system, and the properties of the KEEP scheme may not be exploited due to inadequate conservation at the walls. Thus, investigating the conservative properties of IBM on a conservative scheme like KEEP scheme is essential. In this study, the KEEP scheme and the ghost-cell type IBM were implemented in the compressible BCM framework CUBE [4] and evaluated their performance.

2 Immersed Boundary Method

The ghost-cell type IBM [4-6] is utilized in this study. As shown in Figure 1, the cell adjacent to the wall is denoted as the interface cell (IC). Then, the intersection point, where a perpendicular line drawn from the center of the IC and the wall surface, is defined as the wall point (WP), and the symmetrical point from IC across WP is the virtual point (VP). The value at VP ϕ_{VP} is derived through linear interpolation from the values of the adjacent four cells (eight in 3D), and the values at the IC ϕ_{IC} are determined as follows,

$$\phi_{IC} = \phi_{VP} \quad (\text{Neumann conditions}) \quad (1)$$

$$\phi_{IC} = \phi_{WP} + d \frac{\partial \phi}{\partial n} \quad (\text{Dirichlet conditions}) \quad (2)$$

here, d is the distance between IC and WP, n is the normal vector of wall surface at WP.

Meanwhile, the approaches to the IBM as R. Ghias [5] and C. G. Li[4,6] exhibit variations in the placement of the IC. Ghias' method, called 'ghost-cell IBM', treats the IC as a ghost cell, deriving values from the opposite fluid and IC is calculated by solving the Navier-Stokes equation. Conversely, Li's approach, called 'direct interpolation IBM', acquires the value of IC through interpolation. In the ghost-cell IBM, the conservation law by the Navier-Stokes equation is taken into account for the entire fluid domain. However, this approach requires more memory allocation when both sides of the wall are

considered, because it is necessary to prepare values as fluid and as solid in the same cell.

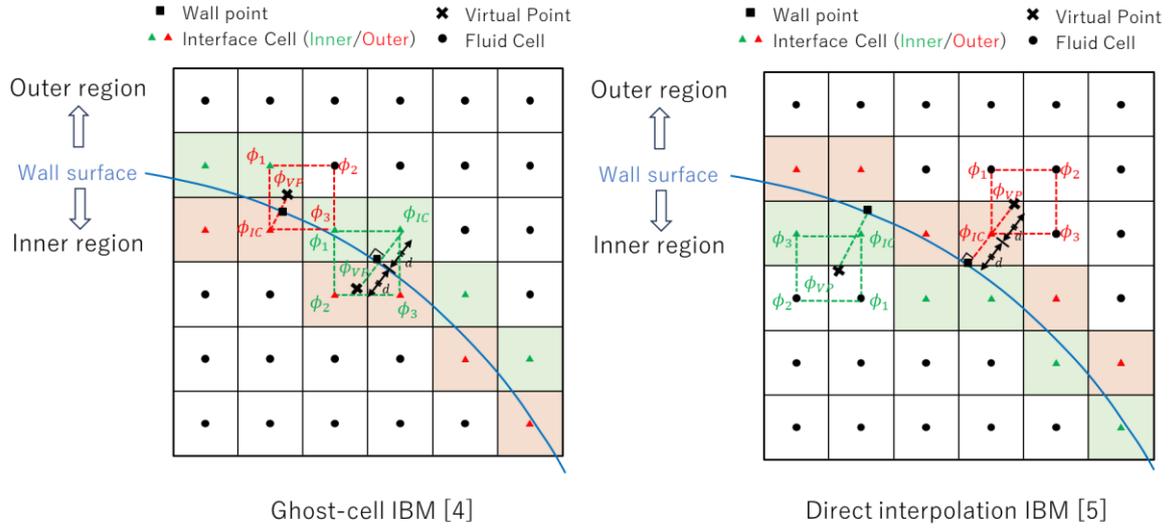


Figure 1 Differences between the two types of IBM

3 Numerical Test

Using the KEEP scheme along two IBMs, the conservation of mass and total energy was assessed by the wall-bounded Taylor-Green vortex (WTGV) [7]. The initial velocity and pressure field are given by

$$u = -M_0 \sin\left(\frac{x}{L}\right) \cos\left(\frac{y}{L}\right) \sin\left(\frac{z}{L}\right) \quad (3)$$

$$v = M_0 \cos\left(\frac{x}{L}\right) \sin\left(\frac{y}{L}\right) \sin\left(\frac{z}{L}\right) \quad (4)$$

$$w = 0 \quad (5)$$

$$p = \frac{1}{\gamma} + \frac{\rho_0 M_0^2}{16} \left(\cos\left(\frac{2x}{L}\right) + \cos\left(\frac{2y}{L}\right) \right) \left(\cos\left(\frac{2z}{L}\right) + 2 \right) \quad (6)$$

with $L(= 1)$ is the reference length, $\gamma(= 1.4)$ is the heat capacity ratio, $M_0(= 0.05)$ is the mach number, and $\rho_0(= 1.0)$ is the initial density. The Reynolds number based on the reference length is $Re = \rho_0 M_0 L / \mu = 1600$. The computational domain is $(2\pi)^3$ box. The TGV has periodic boundaries in each direction, while the WTGV sets up two parallel walls with boundaries at velocity 0. In this case,

$$\begin{aligned} (u, v, w)|_{z=0} &= 0 \\ (u, v, w)|_{z=2\pi} &= 0 \end{aligned} \quad (7)$$

To physically satisfy conservation of mass and energy in the system, neumann boundary conditions are imposed on pressure and density.

$$\frac{\partial \rho}{\partial z} \Big|_{z=0, 2\pi} = 0 \quad (8)$$

$$\frac{\partial p}{\partial z} \Big|_{z=0, 2\pi} = 0 \quad (9)$$

The boundary condition (7)~(9) are modeled by two types of IBMs. The third-order Runge-Kutta method was used for time integration. The computational grids used were 32^3 , 128^3 , and 256^3 cells. The CFL was 0.21 in all cases.

Figure 2 shows the time evolution of mass and total energy of TGV and WTGV. At first, it can be

seen that in normal TGV where all boundaries are periodic, mass and total energy remain unchanged from their initial values at any resolution, indicating that the KEEP scheme is well conserved. However, the direct interpolation IBM shows a significant reduction in mass and a corresponding reduction in total energy in the coarse resolution case of 32^3 cells. This conservation improves as the resolution increases. On the other hand, the ghost-cell IBM conserves mass and total energy even in the low resolutions case of 32^3 cells, showing better conservation than the direct interpolation ghost cell even when the walls are not fully resolved. At the conference, the usefulness of the scheme in other application problems will be discussed.

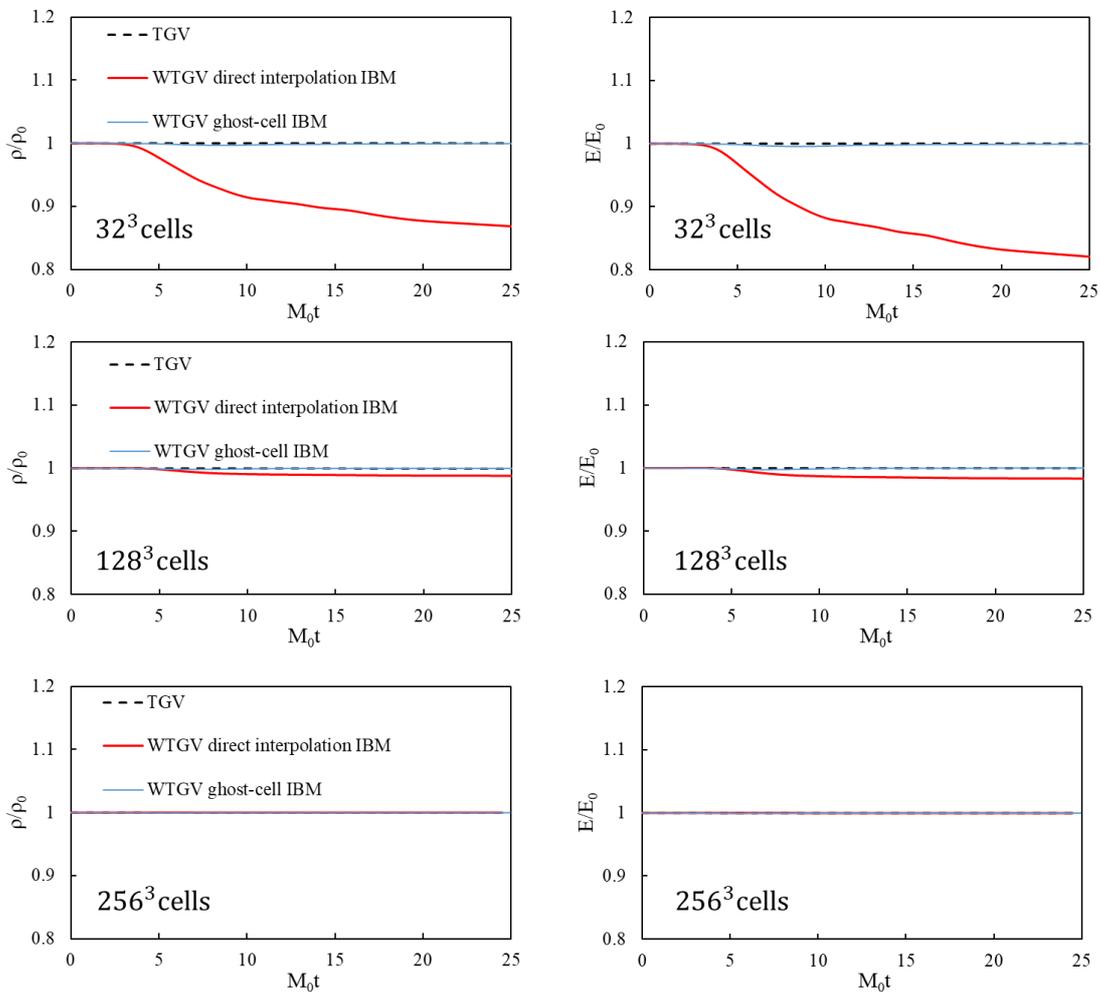


Figure 2 Time evolution of mass(left) and total energy (right) in TGV and WTGV.

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