Oral presentation | Higher order methods

Higher order methods-IV

Tue. Jul 16, 2024 4:30 PM - 6:30 PM Room C

[6-C-04] Locally Divergence-Free Oscillation-Eliminating Discontinuous Galerkin Methods for Ideal MHD Equations

*Wei Zeng[1] (1. Beijing Computational Science Research Center)

Keywords: Ideal compressible MHD equations, Locally divergence-free, OEDG

Twelfth International Conference on Computational Fluid Dynamics (ICCFD12), Kobe, Japan, July 14-19, 2024

ICCFD12-2024-xxxx

Locally Divergence-Free Oscillation-Eliminating Discontinuous Galerkin Methods for Ideal MHD Equations

W. Zeng* and Q. Wang*

Corresponding author: qian.wang@csrc.ac.cn * Mechanics Division, Beijing Computational Science Research Center, Beijing 100193, China

1 Abstract

The ideal magnetohydrodynamics (MHD) equations describe the dynamic behaviors of a perfectly conducting quasi-neutral plasma. Numerical simulation of ideal compressible MHD is challenging, as it needs to be magnetic divergence-free for general cases as well as oscillation-free for cases involving discontinuities. To overcome these difficulties, we propose a locally divergence-free oscillation-eliminating discontinuous Galerkin (LDF-OEDG) method. A set of enriched polynomial basis functions that can automatically satisfy the local divergence-free property [1] are used to approximate the solution on each control volume. The piece-wise LDF polynomials are limited in an oscillation-eliminating (OE) procedure [2] to suppress spurious oscillations near discontinuities. The solutions are updated by using the strong stability preserving Runge-Kutta time integration schemes [3]. The method has the following characteristics: (i) magnetic divergence-free; (ii) stable under normal CFL conditions; (iii) free of characteristic decomposition; (iv) non-intrusive, simple and efficient. The implementation of the LDF-OEDG scheme is described in Algorithm 1. The robustness and accuracy of the proposed method are validated by several benchmark MHD cases, including the vortex problem [4] (see Table 1), the Orszag-Tang problem [5] (see Figure 1), Rotor problem [6] (see Figure 2), etc.

Algorithm 1 LDF-OEDG for ideal compressible MHD equations.

function $\mathbf{U}_\sigma^{n+1}$=LDF-OEDG($\mathbf{U}_\sigma^n$, τ)
  Set $\mathbf{U}_\sigma^{n,0} = \mathbf{U}_\sigma^n$
  Set SSPRK3 coefficients: $c_1 = 1$, $c_1 = \frac{1}{4}$, $c_3 = \frac{2}{3}$
  for $s \leftarrow 1, 3$ **do**
    Surface and volume flux integrals in (1) to compute right-hand-side $\mathcal{T}_f\left(\mathbf{U}_\sigma^{n,s-1}\right)$
    Runge-Kutta stage update $\mathbf{U}_h^{n,s} = (1-c_s)\,\mathbf{U}_\sigma^n + c_s\left(\mathbf{U}_\sigma^{n,s-1} + \tau\mathcal{T}_f\left(\mathbf{U}_\sigma^{n,s-1}\right)\right)$
    OE procedure $\mathbf{U}_\sigma^{n,s} = \mathcal{F}_\tau \mathbf{U}_h^{n,s}$ using the exact damping operator in (2)
    Splitting of $\mathbf{U}_\sigma^{n,s}$ into two parts: the flow field $\mathbf{U}_{\sigma,\mathrm{F}}^{n,s}$ and the magnetic filed $\mathbf{B}_\sigma^{n,s,*}$
    Projection of $\mathbf{B}_\sigma^{n,s,*}$ to obtain a LDF magnetic filed $\mathbf{B}_\sigma^{n,s}$, as in (3)
    Formation of a divergence- and oscillation-free solution $\mathbf{U}_\sigma^{n,s} = \left(\mathbf{U}_{\sigma,\mathrm{F}}^{n,s}, \mathbf{B}_\sigma^{n,s}\right)^{\mathrm{T}}$
  end for
  Update solution $\mathbf{U}_\sigma^{n+1} = \mathbf{U}_\sigma^{n,3}$
end function

Here,

$$\int_K (\mathbf{U}_h)_t\, \phi_K^{(\boldsymbol{\alpha})}\, d\boldsymbol{x} + \oint_{\partial K} \hat{\mathbf{F}}\cdot\mathbf{n}\, \phi_K^{(\boldsymbol{\alpha})}\, ds - \int_K \mathbf{F}\cdot\nabla\phi_K^{(\boldsymbol{\alpha})}\, d\boldsymbol{x} = 0, \quad |\boldsymbol{\alpha}| \le k, \tag{1}$$

$$\mathcal{F}_\tau \mathbf{U}_h = \mathbf{U}_K^{(\mathbf{0})}\phi_K^{(\mathbf{0})}(\boldsymbol{x}) + \sum_{j=1}^{k} \mathrm{e}^{-\tau\sum_{m=0}^{j}\delta_K^m(\mathbf{U}_h)} \sum_{|\boldsymbol{\alpha}|=j} \mathbf{U}_K^{(\boldsymbol{\alpha})}\phi_K^{(\boldsymbol{\alpha})}(\boldsymbol{x}), \tag{2}$$

$$\mathbf{B}_h^{n+1} = \sum_{l=1}^{9} B_K^{(l)}\boldsymbol{\psi}_K^{(l)}, \quad B_K^{(l)} = \frac{\int_K \mathbf{B}_h^{n+1,*}\cdot\boldsymbol{\psi}_K^{(l)}\, d\boldsymbol{x}}{\int_K \boldsymbol{\psi}_K^{(l)}\cdot\boldsymbol{\psi}_K^{(l)}\, d\boldsymbol{x}}. \tag{3}$$

Table 1: Accuracy test results for the vortex problem.

	ρ		ρu_x		B_x		E	
Mesh	L^2-error	Order	L^2-error	Order	L^2-error	Order	L^2-error	Order
16×16	2.12E-03		1.10E-02		1.14E-02		1.50E-02	
32×32	2.59E-04	3.03	5.92E-04	4.22	5.84E-04	4.28	9.88E-04	3.93
64×64	4.45E-05	2.54	5.54E-05	3.42	5.17E-05	3.50	1.23E-04	3.01
128×128	7.29E-06	2.61	7.83E-06	2.82	6.51E-06	2.99	1.86E-05	2.73

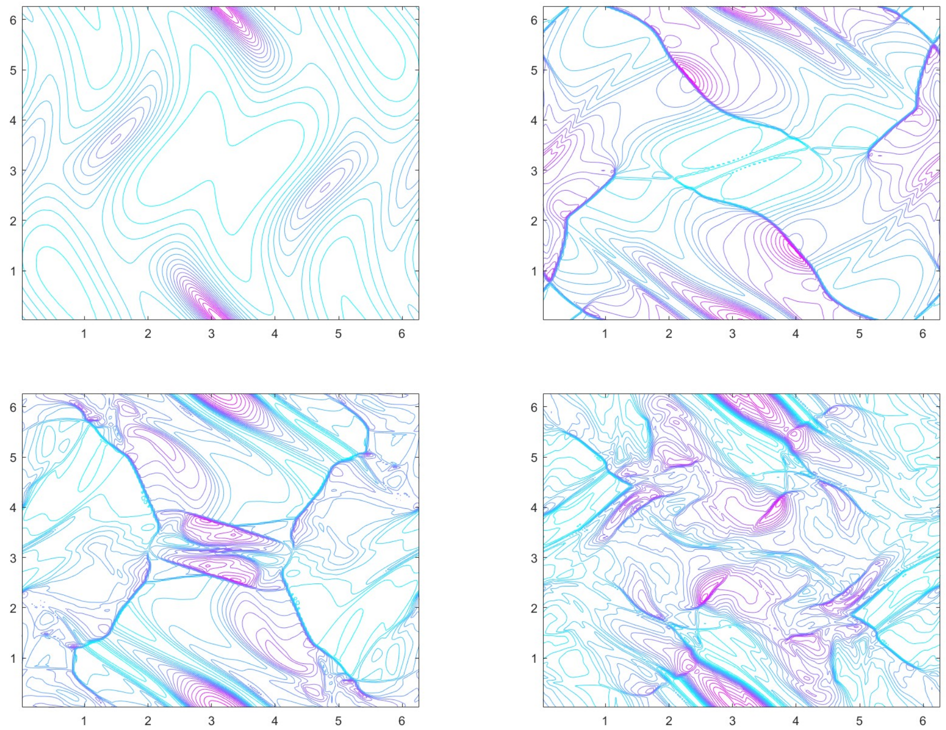

Figure 1: Orszag-Tang problem. Density contours at $t = 0.5$ (top left), $t = 2$ (top right), $t = 3$ (bottom left) and $t = 4$ (bottom right), respectively, computed by LDF-OEDG on a 192×192 mesh.

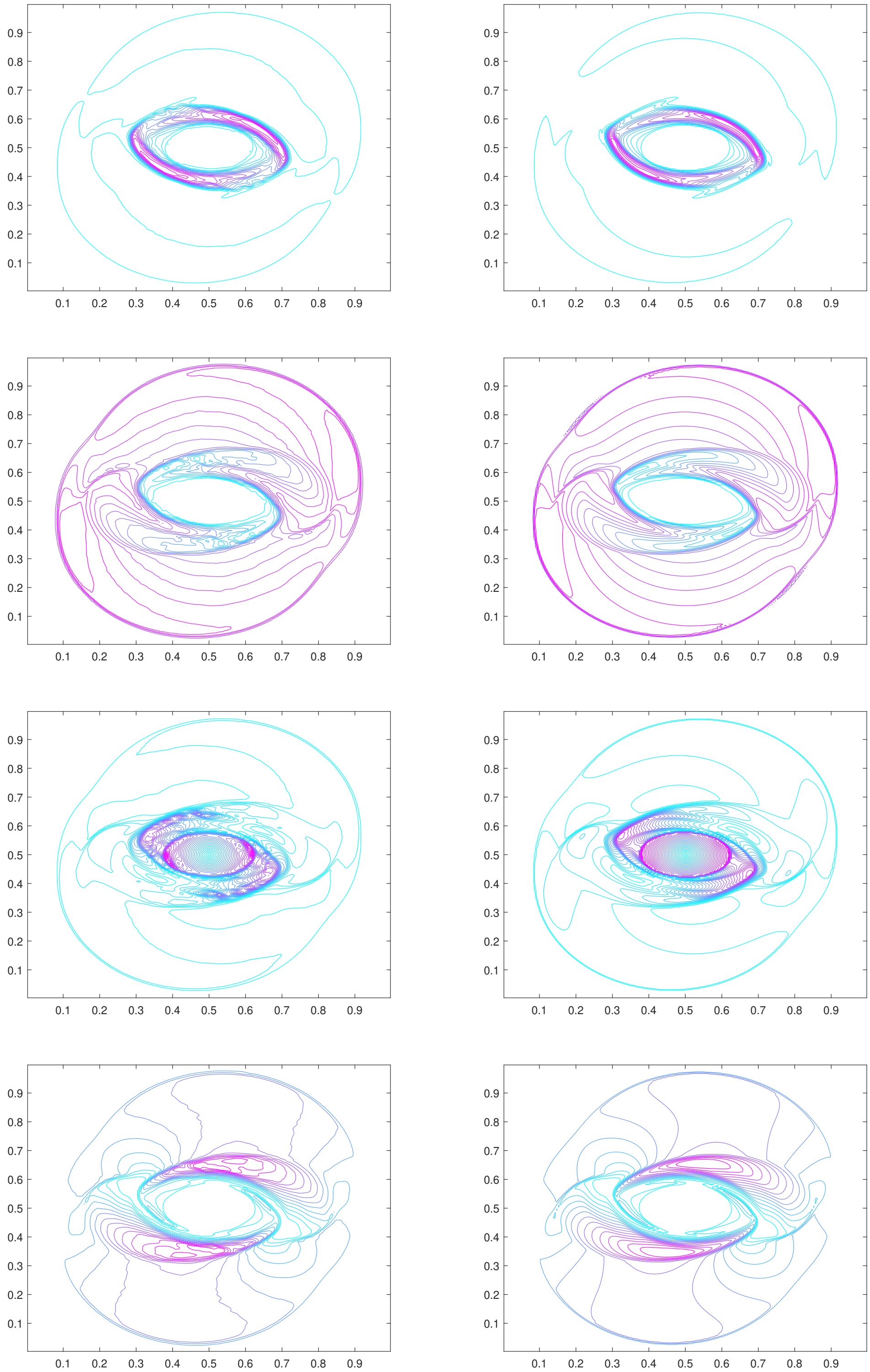

Figure 2: Rotor problem. From top to bottom: contour plots of the density ρ, thermal pressure p, Mach number $\|\mathbf{u}\|_2/c$ and magnetic pressure $\|\mathbf{B}\|_2^2/2$, respectively. The solutions are computed on a 200×200 mesh. Left: LDF-DG with TVB limiter; right: LDF-OEDG.

References

[1] B. Cockburn, F. Li, and C.-W. Shu. Locally divergence-free discontinuous Galerkin methods for the Maxwell equations. *J Comput. Phys.*, 194(2):588–610, 2004.

[2] M. Peng, Z. Sun, and K. Wu. OEDG: Oscillation-eliminating discontinuous Galerkin method for hyperbolic conservation laws. [https://arxiv.dosf.top/abs/2310.04807], 2023.
[3] B. Cockburn and C.-W. Shu. The Runge–Kutta local projection P1-discontinuous-Galerkin finite element method for scalar conservation laws. *ESAIM: Math. Model. Numer. Anal.*, 25:337–361, 1991.
[4] D. S. Balsara. Second order accurate schemes for magnetohydrodynamics with divergence-free reconstruction. *ApJS*, 151(1):149, 2004.
[5] F. Li and C.-W. Shu. Locally divergence-free discontinuous Galerkin methods for MHD equations. *J. Sci. Comput.*, 22:413–442, 2005.
[6] D. S. Balsara and D. Spicer. A staggered mesh algorithm using high order Godunov fluxes to ensure solenoidal magnetic fields in magnetohydrodynamic simulations. *J. Comput. Phys.*, 149:270–292, 1999.