Oral presentation | Turbulent flow

Turbulent flow-I

Tue. Jul 16, 2024 4:30 PM - 6:30 PM Room B

[6-B-01] *A priori* test of a data-driven SGS model considering the multiscale nature of turbulent flows

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Keywords: Turbulent flow, Deep learning, Convolutional neural network

A priori test of a data-driven SGS model considering the multiscale nature of turbulent flows

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Outline

Concept of Data-driven SGS model

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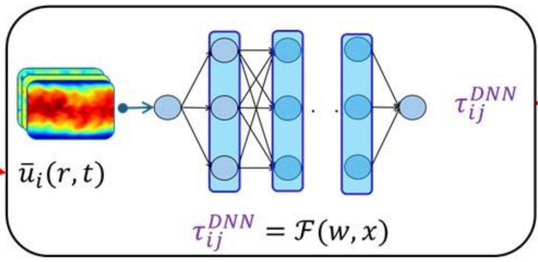
- In recent years, many attempts have been reported to construct data-driven sub-grid scale (SGS) models with deep neural networks (DNN).
- Unlike conventional models, data-driven SGS models are expected to inductively extract subfilter-scale fluctuations and create phenomenon-based models that do not contain artificial approximations or assumptions when trained in appropriate settings.

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(2\nu \bar{D}_{ij} - \tau_{ij}^{DNN}\right)$$



$\hat{y}$: output/prediction
$\mathcal{F}$: model
w: weight
x: input data
r, t: position and time

Ref:
Duraissamy et al. (2019). Annu. Rev. Fluid Mech.

Related work

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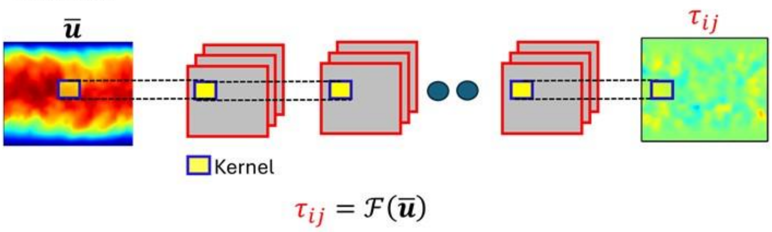
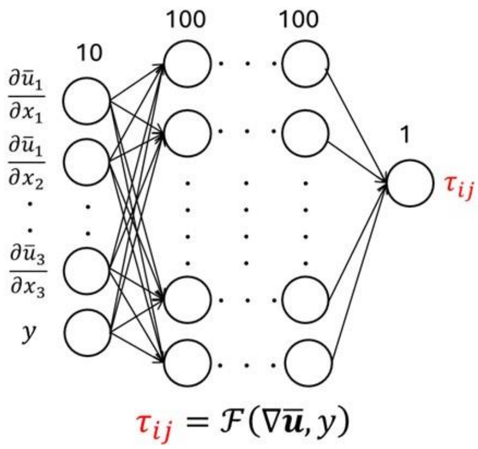
- Data driven SGS model has been deployed based on local approach of multilayer perceptron (MLP)[1,2] and non-local approach of convolutional neural network (CNN) approach[2] showing a good result in predicting residual stress [3,4]
- DNN architecture is critical in data driven SGS model since it involve a large range of different scales of eddies



Data driven SGS based on CNN



Data driven SGS based on MLP

Ref:
1. Gamahara,M, Hattori, Y. (2017). Physical review fluids
2. Park, J and Choi H. (2021). J. Fluid mech.
3. Liu et. al. (2022) AIP Advances
4. Saura, N and Gomez, T. (2023). EPL

Multi-scale CNN-based SGS model

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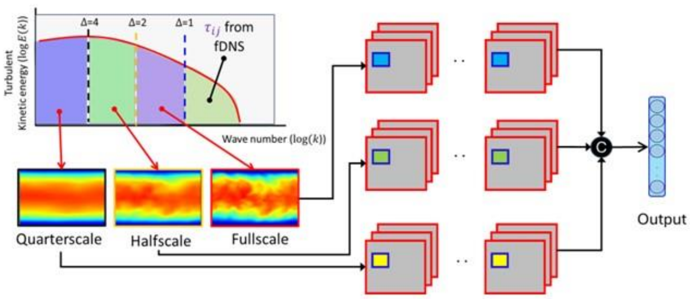
- Multi-scale nature of turbulence vortices → involve a range of vortices scale
- The multi-scale CNN is separating the input to several representation focusing on different scale and progressively encode information from coarser to the finest scale.



Objective
To investigate whether the new multi-scale CNN model can extract features of turbulence vortices of various scales and how the algorithm affects the prediction accuracy of the residual stresses.

Ref:
1. Illaramendi, et. al. (2022). Data centric engineering
2. Fukami, et. al. (2020). Theor. Comput. Fluid Dyn.

Outline

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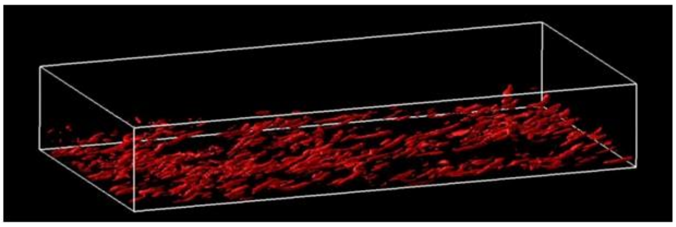
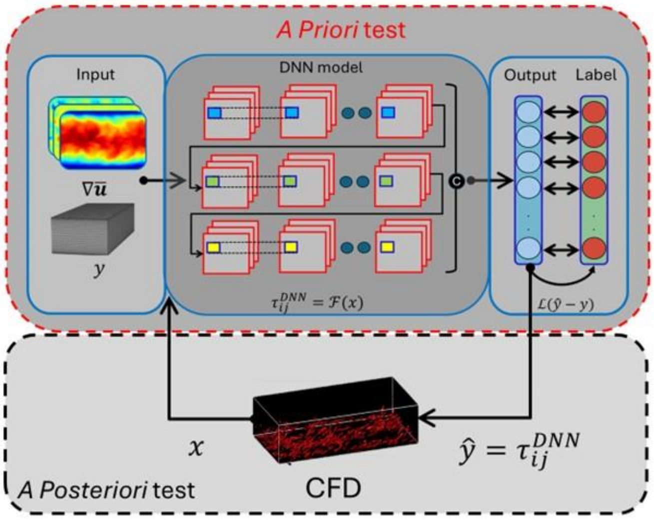
Analysis object & problem setting

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[-\tau_{ij} + \nu\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\right]$$

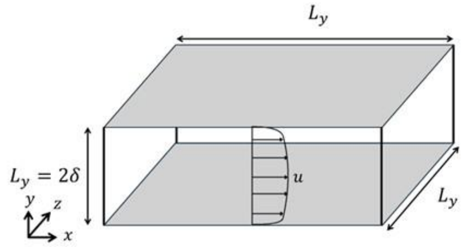


Channel turbulence
friction Reynolds number ($Re_\tau = u_\tau \delta / \nu$) 180



Outline

Dataset preparation: DNS data



Governing Equation

$$\frac{\partial u_i}{\partial x_i} = 0 \; ; \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \delta_{i1} - \frac{\partial p}{\partial x_i} + \frac{1}{Re_\tau} \frac{\partial^2 u_i}{\partial x_i x_j}$$

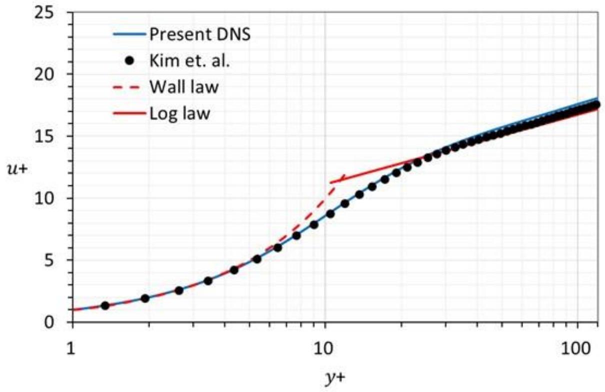
Reynolds number ($Re_\tau = u_\tau \delta / \nu$): 180

Computational setup

Grid arrangement: collocated grid
Time marching: 2nd-order Adams-Bashforth method
Convection & viscous terms: 2nd-order central difference
Coupling of u & p: fractional step method

Computational domain

Variable	Value
$L_x/\delta \times L_y/\delta \times L_z/\delta$	$4\pi \times 2 \times 2\pi$
$\Delta x^+ \times \Delta y^+_{\max} \times \Delta z^+$	$11.8 \times 5.4 \times 7.1$
$N_x \times N_y \times N_z$	$192 \times 128 \times 160$



1. Okabayashi, K. (2016). Journal of Fluid Science and Technology
2. Kim et. al. (1987). Journal of Fluid Mechanics

Dataset preparation: Filtering

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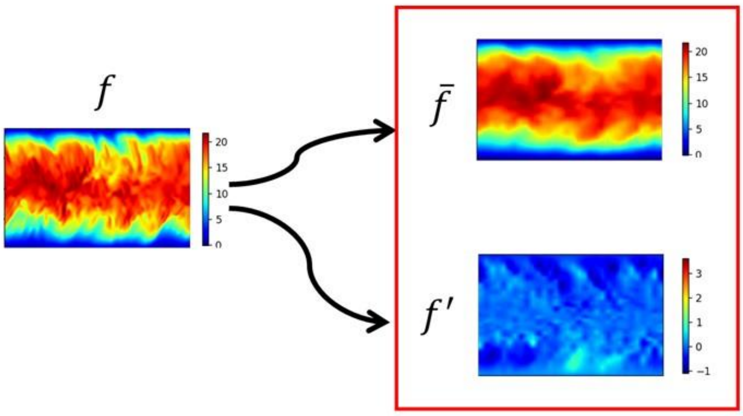
- The dataset for training DNN model is obtained from high fidelity DNS data through filtering process.
- Filtering process will decompose the grid-scale ($\bar{f}$) and subgrid-scale (f') components that match the LES grid. The box filter

$$G(x) = \begin{cases} 1/\Delta & (|x| < \Delta/2) \\ 0 & (|x| > \Delta/2) \end{cases}$$

is used.

Δ: filter size



Filtered DNS (fDNS) data

Outline

Ref:
1. Wang, et. al (2018). Physics of fluid
2. Wu, et. al. (2018). Physical review fluids
3. Gamahara,M, Hattori, Y. (2017). Physical review fluids
4. Prakash et. al. (2022). Comput. Methods. Appl. Mech. Engrg.

The choice of input and output features

- It can be assumed that residual stress may be written as a functional strain and rotation rate tensor as:

$$\tau_{ij}^{DNN} = \mathcal{F}\left(\overline{D}_{ij}, \overline{\Omega}_{ij}\right).$$

- Previous studies[1,2,3,4] found that $\frac{\partial \bar{u}_i}{\partial x_j}$ is the most influential variable rather than $\overline{D}_{ij}$ and $\overline{\Omega}_{ij}$.
- Including $\frac{\partial \bar{u}_i}{\partial x_j}$ as an input, the data-driven SGS model will satisfy Galilean invariance.
- The scalar value of distance from the wall information (y) is provided to give model robustness. Therefore, the data driven SGS model is defined as

$$\tau_{ij}^{DNN} = \mathcal{F}\left(\frac{\partial \bar{u}_i}{\partial x_j}, y\right)$$

- Label data is obtained as $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$.

$\frac{\partial \bar{u}_i}{\partial x_j}$: Velocity gradient tensor
$\overline{D}_{ij}$: Rate-of-strain tensor
$\overline{\Omega}_{ij}$: Rotational rate tensor

Convolutional neural network (CNN)

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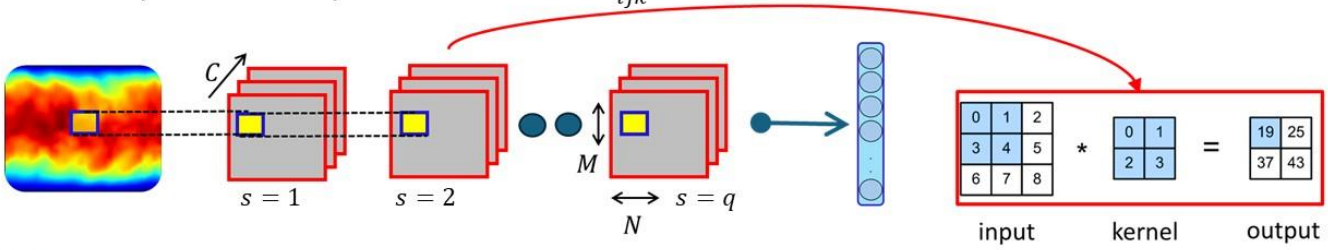
- CNN algorithm is chosen since it provides the advantages of extracting features while preserving location information → provides robustness against spatial invariance.
- To deal with interaction between multiscale phenomena on 3D turbulent flow, 3D CNN algorithm is employed
- The output of each layer are defined as: $x_{ijk}^{(s)}$



Ref:
Liu et. al. (2022) AIP Advances
Goodfellow, et. al. (2016). Deep learning. MIT Press
Brunton et al. (2020). Ann. Review. of. Fluid Mechanics Vol 52
Illaramendi, et. al. (2022). Data centric engineering
Fukami, et. al. (2024). J. Fluid Mech.

$$x_{ijk}^{(s)} = \left(\sum_{c=1}^{C} \sum_{m=0}^{M} \sum_{n=0}^{N} w_{mnkc}^{(s)} x_{i+m-G,j+n-G,c}^{(s-1)} \right)$$

M, N: height and width of filter
C: number of feature map/channel: 16
G: offset/padding
s: layer index
q, k: number of layer and kernel

Ref:
Illaramendi, et. al. (2022). Data centric engineering
Fukami, et. al. (2024). J. Fluid Mech.

Multi-scale CNN

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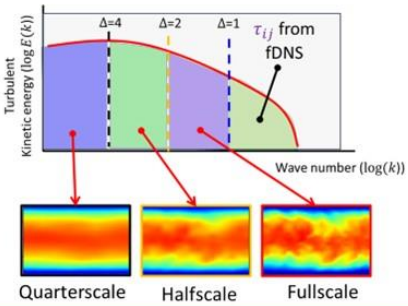
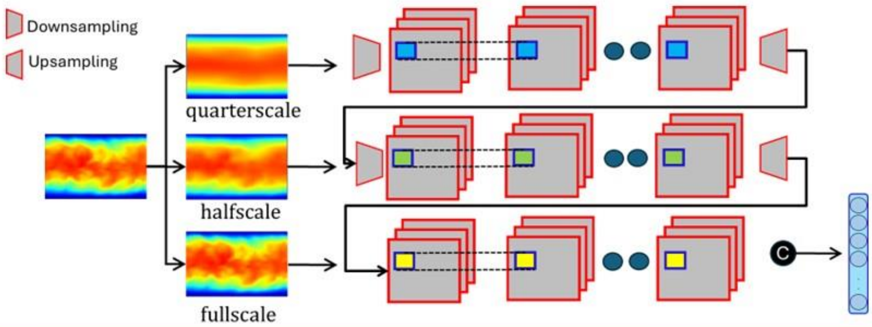
- To accurately approximate the multi-scale nature of turbulence vortices, physical processes between the scales are attempted to be incorporated.
- The input variable will be down-sampled into half and quarter size in which similar with filtering operation. The quarter-scale focuses on largest eddies scale which is the dominant feature while the full-scale retain all eddies scale information. The information from coarsest scale to the finest scale will be progressively encoded ensuring the transfer energy process. The resulting output of each process is concatenated in the final layer





Ref:
Goodfellow, et. al. (2016). Deep learning. MIT Press
Raissi et al. (2019). J. Comput. Phys., 378:686–707
Bose, R., Roy., A. (2024). Engineering Applications of Artificial Intelligence 128

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Loss function (1)

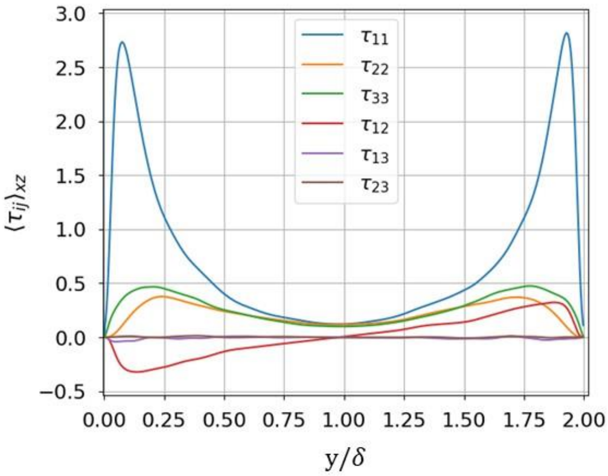
- The loss function consist of data-based loss ($\mathcal{L}_d$), physics constrained loss ($\mathcal{L}_p$), and regularization loss ($\mathcal{L}_r$).

$$\mathcal{L} = \mathcal{L}_d + \mathcal{L}_p + \mathcal{L}_r$$

- The average value of τ_{13} and τ_{23} are very low, making the nonlinear regression of DNN model hard to be reconstructed.
- The constant β is introduced to magnify the influence of τ_{13} and τ_{23}. Therefore, $\mathcal{L}_d$ is described as:

$$\mathcal{L}_d = \sum_{j=1}^{6}\left(\frac{\beta_j}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)\right)$$

y_i: label/truth data (τ_{ij})
$\hat{y}$: output/predicted data



The average value of τ_{ij} over stream and spanwise direction w.r.t channel height

Ref:
Goodfellow, et. al. (2016). Deep learning. MIT Press
Raissi et al. (2019). J. Comput. Phys., 378:686–707
Bose, R., Roy., A. (2024). Engineering Applications of Artificial Intelligence 128

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Loss function (2)

- To satisfy the physical condition, physics constraint loss function ($\mathcal{L}_p$) is introduced by adding the physical data of filtered velocity.
- Regularization loss ($\mathcal{L}_r$) is applied to avoid overfitting. Thus, the total loss function is defined as:

$$\mathcal{L} = \mathcal{L}_d + \mathcal{L}_p + \mathcal{L}_r$$

$$\mathcal{L} = \sum_{j=1}^{6}\left(\frac{\beta_j}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)\right) + \sum_{j=1}^{6}\left(\frac{1}{N}\sum_{i=1}^{N}\theta(y_{2i} - \hat{y}_i)\right) + \lambda\frac{1}{N}\sum_{i=1}^{N}w_i^2$$

$\mathcal{L}_d$: Loss data-based
$\mathcal{L}_p$: Loss physics-based
$\mathcal{L}_r$: Loss regularization
w: weight
x: input
β, θ: constant
y_i: label/truth data (τ_{ij})
$\hat{y}$: output/predicted data
λ: regularization constant

y_{2i} is calculate from fDNS data, denote as $y_{2i} = \overline{u_i u_j} - \overline{u}_i\overline{u}_j$
Whereby in the calculation, the data of velocities will be embedded, not τ_{ij}

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Outline

Architecture comparison

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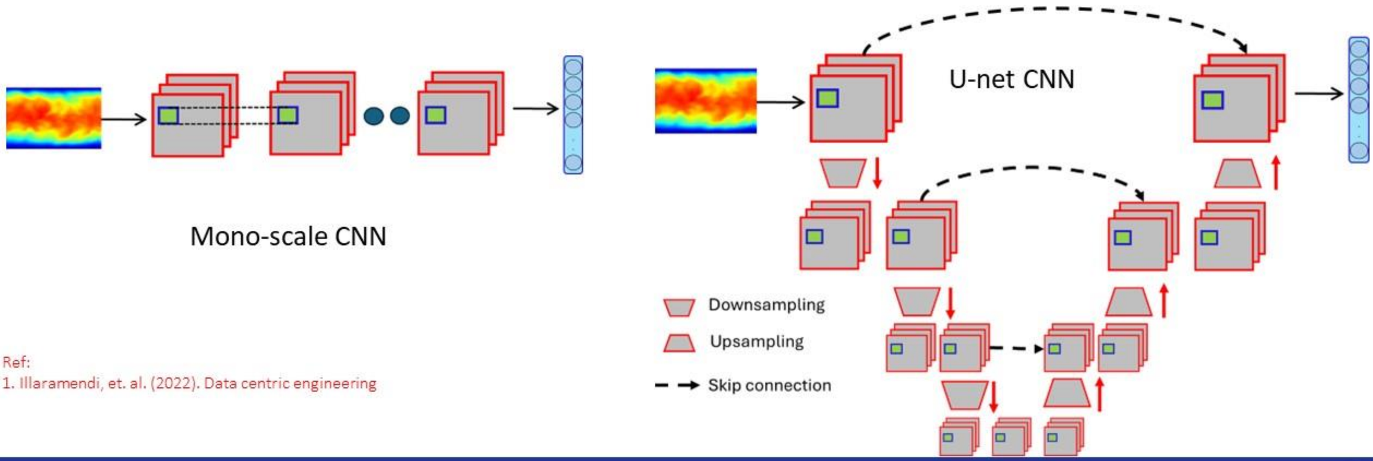
- To investigate the performance of multiscale model, the comparison between the conventional CNN ("mono-scale CNN") and U-net[1] is carried out.



Ref:
1. Illaramendi, et. al. (2022). Data centric engineering

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Outline

Ref: Gholamy, A., et. al. (2018) Why 70/30 or 80/20 Relation between Training and Testing Sets: A Pedagogical Explanation. Departmental Technical Reports (CS)

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Training and test

- **Dataset for training and test**
  - Input data: field of $\frac{\partial \bar{u}_i}{\partial x_j}$, y (distance from walls)
  - Label data: field of $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$
  - Amount of input: 10,000 instantaneous fDNS data (out of $0 \leq t \leq 120$). 8,000 for training data and 2,000 for test data.

- **Correlation coefficient**

$$CC[\tau_{ij}^{fDNS}, \tau_{ij}^{p}] = \frac{\langle (\tau_{ij}^{fDNS} - \langle \tau_{ij}^{fDNS} \rangle)(\tau_{ij}^{p} - \langle \tau_{ij}^{p} \rangle) \rangle}{\sqrt{\langle ((\tau_{ij}^{fDNS} - \langle \tau_{ij}^{fDNS} \rangle)^2 \rangle} \sqrt{\langle ((\tau_{ij}^{p} - \langle \tau_{ij}^{p} \rangle)^2 \rangle}}$$

τ_{ij}^{p} : Predicted data
τ_{ij}^{fDNS} : Label data
$\langle . \rangle$: domain average

Correlation coefficient (1)

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- High correlation value (averaged in the : spanwise, streamwise, and wall normal direction) implies successful learning.
- Multi-scale CNN provided most successful as CC value exceeds 0.8 for all τ_{ij} component.
- In mono-scale CNN, CC < 0.8 on τ_{22} and τ_{23} component while only τ_{11} and τ_{12} component are yielded CC > 0.8 for U-net.
- This result provides that multi-scale CNN predicts the residual stress more accurate than other models.

$CC[\tau_{ij}^{fDNS}, \tau_{ij}^{p}]$	τ_{11}	τ_{22}	τ_{33}	τ_{12}	τ_{13}	τ_{23}	CC average
Multi-scale CNN	0.931	0.860	0.886	0.913	0.881	0.803	0.879
Mono-scale CNN	0.900	0.793	0.842	0.867	0.832	0.723	0.826
U-net	0.869	0.756	0.797	0.822	0.776	0.659	0.780

τ_{ij}^{fDNS}: fDNS data
τ_{ij}^{p}: predicted value

Correlation coefficient (2)

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- τ_{11} is the easiest component to resolve as it given higher CC value for all model.
- The difficulties to resolve τ_{13} and τ_{23} are noted on Refs.[1,2,3]. By applying present loss function, it helps the DNN model to effectively resolved τ_{13} and τ_{23}.
- Present result has a better CC value than Refs.[1,2,3], yielding that physical constrain loss function help the model to increase the correlation value.

$CC[\tau_{ij}^{fDNS}, \tau_{ij}^{p}]$	τ_{11}	τ_{22}	τ_{33}	τ_{12}	τ_{13}	τ_{23}	CC average
Multi-scale CNN	0.931	0.860	0.886	0.913	0.881	0.803	0.879
Mono-scale CNN	0.900	0.793	0.842	0.867	0.832	0.723	0.826
U-net	0.869	0.756	0.797	0.822	0.776	0.659	0.780

τ_{ij}^{fDNS}: fDNS data
τ_{ij}^{p}: predicted value

Ref:
1. Gamahara,M, Hattori, Y. (2017). Physical review fluids
2. Park, J and Choi H. (2021). J. Fluid mech.
3. Bose, R., Roy., A. (2024). Engineering Applications of Artificial Intelligence 128

Correlation coefficient (3)

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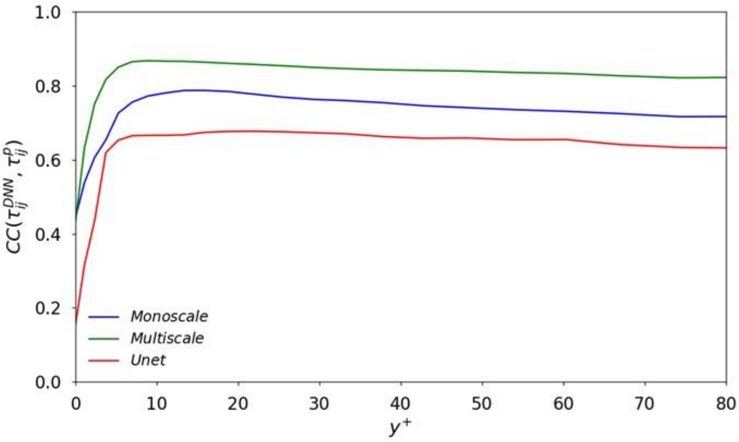
- The averaged CC value over span and streamwise w.r.t y^+ is plotted.
- The result shows the superiority of multi-scale CNN on achieving better result than that of mono-scale CNN and U-net.
- In general, all models provide almost constant CC value on $y^+ > 5$. This constant result can be regarded as the ability of DNN model to effectively predict the residual stress in a whole domain and will not be degraded in specific region.

Ref:
1. Gamahara,M, Hattori, Y. (2017). Physical review fluids
2. Bose, R., Roy., A. (2024). Engineering Applications of Artificial Intelligence 128



τ_{ij}^{fDNS}: fDNS data
τ_{ij}^{p}: predicted value

Correlation coefficient (4)

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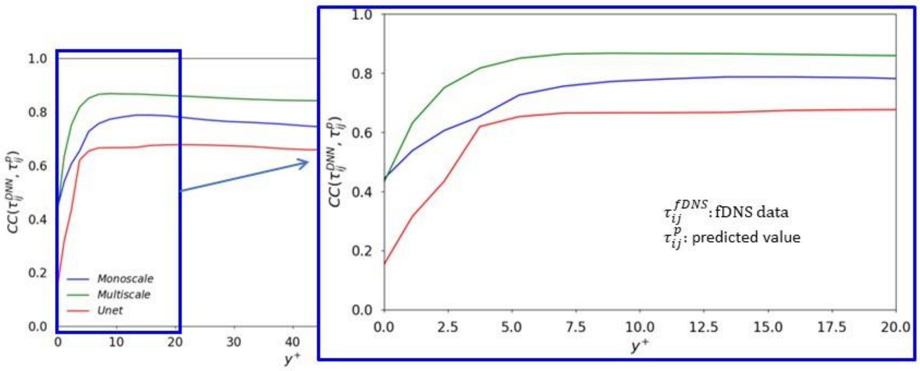
- On the viscous sublayer region, $y^+ < 5$, all model correlation was decreased especially near the wall.
- This results are in contrast with local approach of Refs.[1,2,3], meaning that local approach MLP has a better ability to predict near the wall since it maps the information locally. Meanwhile, non-local CNN approach is given more robust result on the whole domain.



Ref:
1. Gamahara,M, Hattori, Y. (2017). Physical review fluids
2. Bose, R., Roy., A. (2024). Engineering Applications of Artificial Intelligence 128
3. Liu et. al. (2022) AIP Advances

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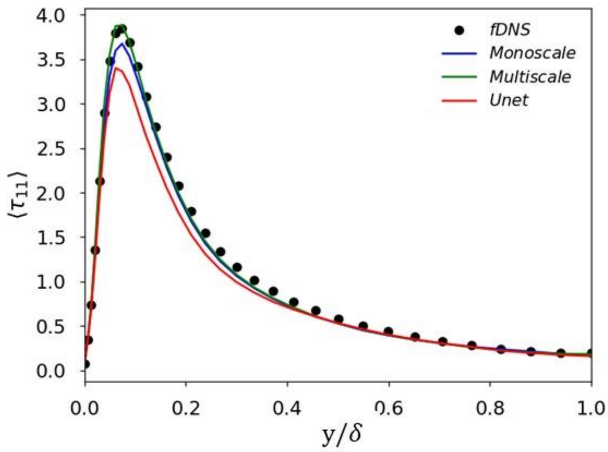
Wall-normal distribution of τ_{ij}

- The wall-normal distribution and the averaged τ_{ij} is investigated.
- The τ_{11} predicted from the multi-scale CNN is obviously closer to fDNS compared to the conventional mono-scale CNN and U-Net.
- By having higher average value, τ_{11} is considerably easier component to be learned by nonlinear regression model of data-driven model.



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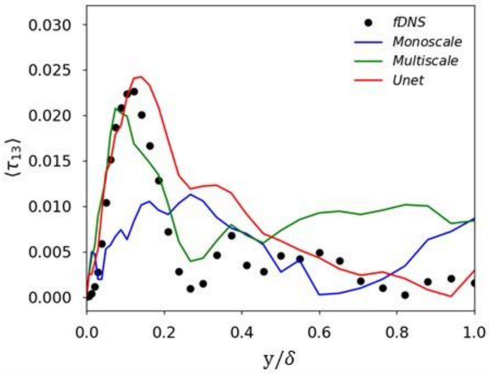
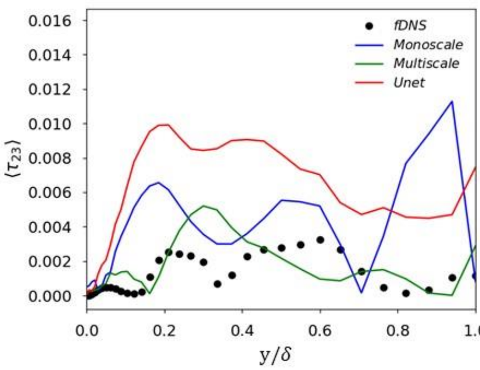
Wall-normal distribution of τ_{ij}

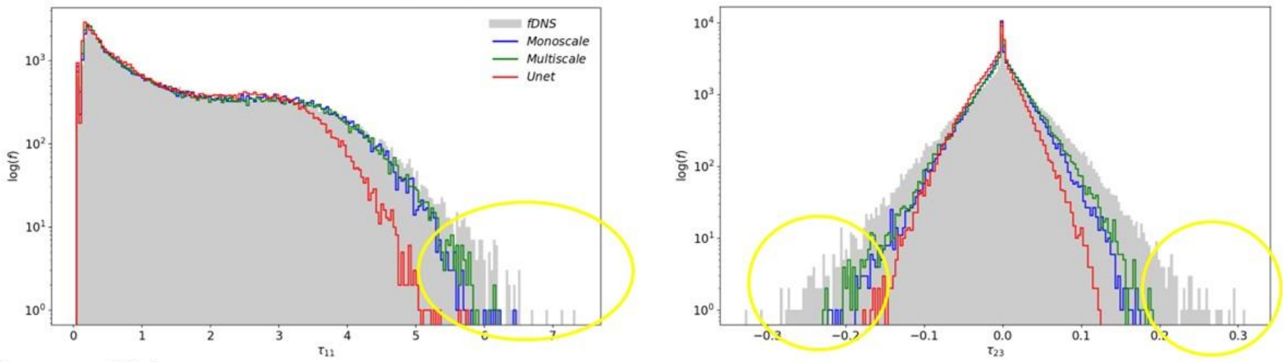
- Conversely, due to the small average value, τ_{13} and τ_{23} are difficult to approximate.
- Both mono-scale and U-net algorithms overpredict the τ_{13} and τ_{23}.
- Meanwhile, the multi-scale CNN has been successfully predicting those values adequately.
- This result showed that the ability of multi-scale on extracting small value or small scales interaction.





Model performance on value distribution

The frequency distribution of the prediction and fDNS.



f: frequency of data τ_{ij}

Overall, all models have been struggling to predict 'outliner' data shown on yellow circle, indicating the limitation of model. However, compared to mono-scale and U-net, multi-scale is considerably better as it can resolve τ_{11} and low value of τ_{23} adequately. It yielded that multi-scale is more effectively to capture and resolve the dynamics complexity of turbulent field

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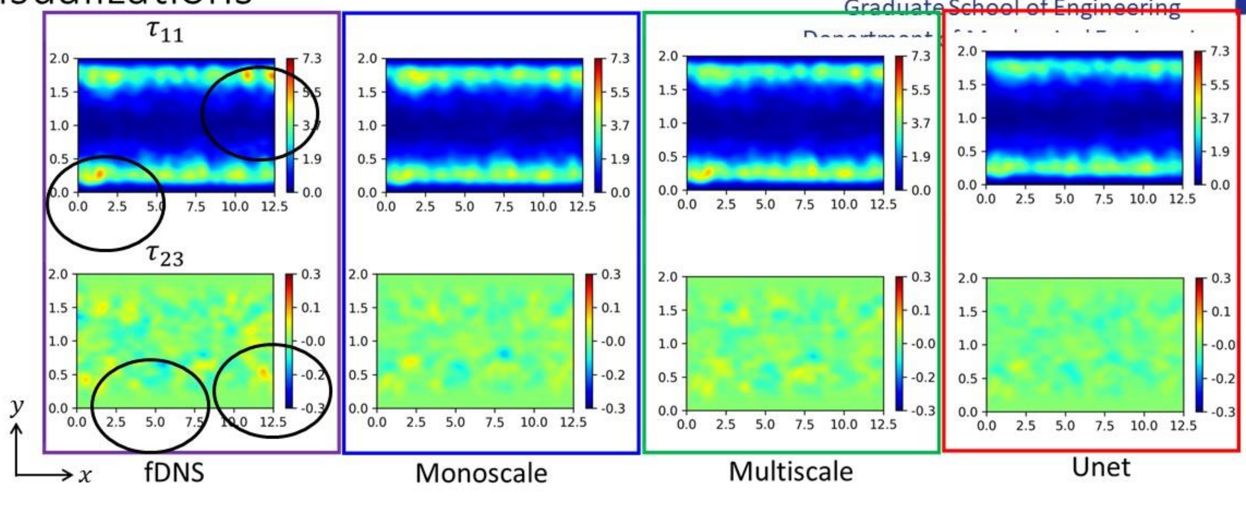
Visualizations



Spatial distribution of τ_{11} and τ_{23} are depicted above. It shows that multi-scale can resolve residual stress accurately on high value of τ_{11}. On the low value of τ_{23}, multi-scale has slightly outperformed mono-scale and U-net. Both mono-scale and U-net show relatively similar result on τ_{11} and struggle in τ_{23}

Discussion

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- The multi-scale model outperforms both the mono-scale and U-net models in resolving τ_{ij}, demonstrating superior capability in extracting important flow features.
- This model is accurately constructing τ_{ij} within different regions, including the viscous sublayer, viscous wall, and outer layer, indicating its robustness in capturing varied flow dynamics while maintaining high CC value
- The investigation of τ_{11} and τ_{23} describes the multi-scale model's proficiency in resolving both large and small scales, highlighting its effectiveness in capturing the complete spectrum of turbulent structures.

Discussion

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- The progressive encoding of features from coarser to finer scales facilitates the incorporation of the energy transfer process between scales, resembling the energy cascade mechanism in turbulent flows.
- This step is crucial as it ensures that the multi-scale model effectively captures the intricate details of energy distribution across scales.
- By extracting features from large, intermediate, and small eddies, the multi-scale model provides the DNN with comprehensive information, enabling it to construct accurate nonlinear regression models for resolving τ_{ij}.

Outline

Conclusion

- The multi-scale model outperforms mono-scale and U-net models in resolving τ_{ij}, effectively extracting key flow features across the viscous sublayer, viscous wall, and outer layer regions.
- By progressively encoding features from coarser to finer scales, it captures the energy transfer process, akin to the energy cascade, ensuring comprehensive detail retention for accurate predicting residual stress.
- This approach enhances the model's ability to capture both large and small scales feature, demonstrating its proficiency in non-linear regression model of DNN.

Acknowledgment

- This work was partly achieved through the Research Proposal-based Use of the Large-scale Computer System at the Cybermedia Center, Osaka University.
- This work was financially supported by JSPS KAKENHI Grant No. JP22K03925 and a Grant from Beasiswa Pendidikan Indonesia (BPI)/ LPDP (the Indonesian Endowment Fund for Education, Ministry of Finance of Republic of Indonesia)

Thank you