[6-A-03] A generic balanced-force algorithm for multiphase flows with moving bodies on polyhedral unstructured grids

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A generic balanced-force algorithm for multiphase flows with moving bodies on polyhedral unstructured grids

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Contents



Challenges of multiphase fluid-structure interaction (FSI) simulations with the arbitrary Lagrangian-Eulerian (ALE) method

- Large fluid density ratio (usually 10³)
- Severe deformation and distortion of the unstructured grid
- Strong nonlinearity during the fluid-structure coupling



(a) Filming of ship navigation







The survey of th



- Density ratio $ho_{
 m H}/
 ho_{
 m L}=10^3$
- Non-orthogonal unstructured grid in the entire domain







Balanced-force algorithm

- Devise a specific discretization to **enforce the exact balance** between pressure gradient and external forces
- Francois et al. (J. Comput. Phy. 213 (2006) 141-173) proposed a balanced-force algorithm on structured grids



Journal of Computational Physics Volume 213, Issue 1, 20 March 2006, Pages 141-173

A balanced-force algorithm for continuous and sharp interfacial surface tension models within a volume tracking framework



Achieve force balance by identical discretization of gradient operators at the same location ?



Cited by (687)

Three dimensional interface normal prediction for Volume-of-Fluid method using artificial neural network 2024, European Journal of Mechanics, B/Fluids

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On polytope intersection by half-spaces and hyperplanes for unsplit geometric volume of fluid methods on arbitrary grids 2024, Computer Physics Communications

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A locally redistributed level-set method for numerical simulation of thin interface structures

2024, Computers and Fluids

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A robust phase-field method for two-phase flows on unstructured grids 2024, Journal of Computational Physics



M STATULI II





Defect of conventional balanced-force algorithm (J. Comput. Phy. 213) (2006) 141-173)

- can **NOT** achieve force balance on non-orthogonal grids and generate severe spurious currents
- Spurious velocity escalates with larger non-orthogonality angle and higher fluid density





Objective: Balanced-force algorithm for fluid-structure interaction (FSI)

- Propose a generic balanced-force (GBF) algorithm for multiphase flows on unstructured grids
- Employ the GBF algorithm for multiphase FSI problems to enhance the stability and accuracy of simulations



STATULE I

Soverning equations in the ALE description $ \frac{\partial \phi}{\partial t} + \nabla \cdot (\phi(\mathbf{u} - \mathbf{u}_g)) = \phi \nabla \cdot \mathbf{u}, \\ \nabla \cdot \mathbf{u} = 0, \\ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho(\mathbf{u} - \mathbf{u}_g) \otimes \mathbf{u}) = (\nabla \cdot (\mu \nabla \mathbf{u}) + \nabla \mathbf{u} \cdot \nabla \mu) - \nabla p - (\mathbf{g} \cdot \mathbf{x})(\rho_H - \rho_L)\nabla \phi + \sigma_K \nabla \phi. $ where ϕ (\mathbf{x} , t) represents the volume fraction separating two immiscible fluids, $\mathbf{u} = (u, v, w)$ is the velocity field, p the dynamic pressure, ρ the fluid density and μ the fluid dynamic viscosity. \mathbf{g} is the gravitational acceleration, σ the surface tension coefficient and κ the curvature of fluid interface. \mathbf{u}_g is the grid velocity. The material properties of fluids: $\rho = \rho_H \phi + \rho_L (1 - \phi)$ $\mu = \mu_H \phi + \mu_L (1 - \phi)$ where subscripts H and L represent heavy and light fluid respectively.	Numerical methods
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Numerical methods



Solution procedure

step n + 1 and calculate the grid velocity \mathbf{u}_{g}^{n+1} . (1) Calculate the dynamics and displacement of moving body with 6 DOF motion equations. Update the grid to time

(2) Solve the advection equation of mass and convection equation of momentum simultaneously:

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= -\nabla \cdot \left(\phi \left(\mathbf{u} - \mathbf{u}_{g}^{n+1} \right) \right) + \phi \nabla \cdot \mathbf{u}, \\ \frac{\partial \rho \mathbf{u}}{\partial t} &= -\nabla \cdot \left(\rho \mathbf{u} \otimes \left(\mathbf{u} - \mathbf{u}_{g}^{n+1} \right) \right) + (\nabla \cdot (\mu \nabla \mathbf{u}) + \nabla \mathbf{u} \cdot \nabla \mu), \end{aligned}$$

to obtain ϕ^{n+1} and \mathbf{u}^* , and update ρ^{n+1} and μ^{n+1} with ϕ^{n+1} .

(3) Solve the following pressure Poisson equation for pressure field p^{n+1} :

$$\nabla \cdot \left(\frac{1}{\rho} \nabla p^{n+1}\right) = \frac{\nabla \cdot \mathbf{u}^*}{\Delta t} - \nabla \cdot \left(\frac{\mathbf{g} \cdot \mathbf{x}}{\rho} (\rho_{\rm H} - \rho_{\rm L}) \nabla \phi^{n+1}\right) + \nabla \cdot \left(\frac{\sigma \kappa}{\rho} \nabla$$

where the gravity and surface tension forces are incorporated to derive a balanced-force formulation.

(4) Correct the intermediate velocity \mathbf{u}^* to \mathbf{u}^{n+1} by the projection step.

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\frac{\mathbf{g} \cdot \mathbf{x}}{\rho} (\rho_{\rm H} - \rho_{\rm L}) \nabla \phi^{n+1} + \frac{\sigma \kappa}{\rho} \nabla \phi^{n+1} - \frac{1}{\rho} \nabla p^{n+1}$$

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The updated velocity field \mathbf{u}^{n+1} should satisfy the divergence-free condition.





Numerical methods
Present balanced-force formulation
Express the averaged face gradient of pressure
$$\overline{p}_{ij}$$
 into the sum of external forces and a novel pressure correction term $\nabla \hat{p}_i$:
 $\overline{p}_{ij} = (-(\mathbf{g} \cdot \mathbf{x}_{ij})(\rho_{\mathrm{H}} - \rho_{\mathrm{L}}) + \sigma \kappa_{ij})\overline{\phi}_{ij} + \overline{p}_{ij}} = (-(\mathbf{g} \cdot \mathbf{x}_{ij})(\rho_{\mathrm{H}} - \rho_{\mathrm{L}}) + \sigma \kappa_{ij})\overline{\phi}_{ij}^{\mathrm{T}} + \overline{p}_{ij}$
 $\overline{p}_{ij}^{\mathrm{T}} = (-(\mathbf{g} \cdot \mathbf{x}_{ij})(\rho_{\mathrm{H}} - \rho_{\mathrm{L}}) + \sigma \kappa_{ij})\overline{\phi}_{ij}^{\mathrm{T}} + \overline{p}_{ij}$
 $\nabla alculate \nabla \hat{p}_{i}^{\mathrm{T}}$ with the Eq. (6), and linear interpolate to $\overline{p}_{ij}^{\mathrm{T}}$
 $\nabla p_{i}^{\mathrm{T}} = \left(\sum_{j=1}^{J} \Gamma_{ij} \otimes \mathbf{n}_{ij}\right)^{-1} \cdot \sum_{j=1}^{J} \left((\nabla p_{ij}^{\mathrm{T}} \cdot \mathbf{n}_{ij}) - (-(\mathbf{g} \cdot \mathbf{x}_{ij})(\rho_{\mathrm{H}} - \rho_{\mathrm{L}}) + \sigma \kappa_{ij})(\nabla \phi_{ij}^{\mathrm{H}+1} \cdot \mathbf{n}_{ij})\right) \Gamma_{ij}$ (6)
 \cdot Present semi-implicit pressure Poisson equation:
 $\sum_{j=1}^{J} \alpha_{ij}(p_{j}^{\mathrm{T}+1} - p_{i}^{\mathrm{T}+1}) = \sum_{j=1}^{J} (-(\mathbf{g} \cdot \mathbf{x}_{ij})(\rho_{\mathrm{H}} - \rho_{\mathrm{L}}) + \sigma \kappa_{ij})\alpha_{ij}(\phi_{j}^{\mathrm{T}+1} - \phi_{i}^{\mathrm{T}+1})$
Implicit cutral difference term
 $-\sum_{j=1}^{J} p_{ij} \cdot \overline{p}_{ij}^{\mathrm{T}}$ $+ \sum_{j=1}^{J} |\Gamma_{ij}|(\mathbf{u}_{ij}^{*} \cdot \mathbf{n}_{ij})$
Explicit non-orthogona correction term



Numerical methods



Numerical schemes

- Based on finite volume method for polyhedral unstructured grids
- Interface capturing scheme: THINC/QQ (The Tangent of Hyperbola for INterface Capturing method with Quadratic surface representation and Gaussian Quadrature)
- Velocity discretization scheme: FVMS3 (The Finite Volume method based on Merged Stencil with 3rd-order reconstruction)
- MLP (Multi-dimensional Limiting Process) and BVD (Boundary Variation Diminishing)
- Generic balanced-force algorithm
- Time scheme: Third-order TVD Runge-Kutta scheme
- 6DoF body motions: Newmark's method



Numerical tests

surface tension force on unstructured grids unbalance between pressure gradient and Suppress the spurious velocity due to



③ 3D static drop in equilibrium

Equation: $\nabla p = \sigma \kappa \nabla \phi$ Theoretical solutions: Surface tension coefficients: $\sigma = 73$ Fluid density ratio: $\rho_{\rm H}/\rho_{\rm L} = 10^6$ Computational domain: $[0, 8]^3$ Drop radius: R = 2

$$\mathbf{u} = \mathbf{0}$$
$$\Delta p = 2\sigma/R$$

grid A

grid B

grid C











Numerical tests predictions for practical applications Provide more accurate and robust



- Two wave energy converters interaction with regular waves
- Regular wave: *H* = 0.074 m, *T* = 1.26 s
- $F_{\text{PTO}} = -\mu F_{\text{Spring}} \operatorname{sign}(w(t)) = -\mu (4k_{\text{Spring}} dl) \operatorname{sign}(w(t))$

 $F_{\text{Bearing}} = -\mu abs(F_{\text{Surge}}(t))sign(w(t))$



-0.04 40

42

44

46

48

50

t(s)

Conclusion



A generic balanced-force algorithm for FSI problems

- considerably suppresses the spurious velocity due to the mesh non-orthogonality
- for multiphase flows in moving mesh configurations
- effectually restrains the instability and divergence induced by large mesh

deformation and non-linearity during FSI simulations

offers a promising platform to provide more accurate and robust predictions for practical multiphase flow simulations involving strong fluid-structure interactions

https://doi.org/10.1016/j.jcp.2023.112010

http://dx.doi.org/10.2139/ssrn.4668121

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Thanks for your attention!

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