Oral presentation | Data science and AI Data science and AI-IV Fri. Jul 19, 2024 10:45 AM - 12:45 PM Room C

[13-C-03] A low-dissipation reconstruction scheme for compressible single- and multi-phase flows based on the artificial neural network

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Keywords: Compressible flows, Single- and multi-phase fluids, Artificial neural networks



- Introduction
- MUSCL-THINC-BVD
- Deep-MUSCL-THINC-BVD(DeepMTBVD)
- Numerical Method
- Numerical Result
- Conclusion

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Background



Compressible fluid dynamics is

one of the most active and challenging research areas in CFD, with applications in aerospace engineering, the automotive and the defense industries, etc.

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Numerical method of resolving discontinuty in compressible flow

- Multi-moment Constrained finite Volume method(MCV)
- High order Discontinuous Galerkin (DG)
- Flux Reconstruction (FR)/Correction Procedure via Reconstruction (CPR)
- Conventional Finite Volume Method (FVM)
- Deep Learning Method (PINNs, DeepONet)

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Reconstrc	ution in FVM				
Piecewise Pros: Simple imple Cons: Low accurace Excessive di TVD Sche Pros: Second orde Simple imple Cons: Excessive di extrema	e constant ementation cy ssipation eme (MUSCL) er in smooth region ementation sspation especially around	d discontinuities and	(W/T)ENO a Pros: High-order accura Parallelizable Robust Cons: Wide stencils Complexibility of	nd its variants acy and resolution choosing admissible stencils	
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Boundary variation diminishing (BVD) scheme

MUSCL-THINC-BVD

Introduction

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BVD Scheme		
 Discontinous region: THINC s Gibbs phenomenon is inevitable Sigmoid functions performs bette Smooth region: Polynomial-base 	cheme BVD scheme in struct One-stage: Z. Sun, 20: Two-stage: X. Deng, 2 Multi-stage: X. Deng, 2 ased scheme	c tured grid: 16, X.Deng 2018a, Z. Hou, 2021, C. Pan, 2024 019 2020a, X. Deng, 2020b, X. Deng, 2022c
 High accuracy in smooth region High resolution of discontinuty, mat Robustness and adaptivity Prepare all candidate reconstruction 	BVD scheme in unst serial interface Single-phase flow: X. I Multi-phase flow: L. Cl Deep	ructured grid: Deng, 2017a, X.Deng, 2017b heng, 2021, L. Cheng, 2022 Learning
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Deep Learr	Deep Learning Method								
Directly :DeepON	Solving PDEs: let		 Supplement Trouble cell d 	to traditional m etector	ethods:				
 FNO 			 Flow structure 	e detector					
 PINNs 			 Reconstruction based on neural network 						
 Deep Rit 	z Method		Keeps framework of	f codes					
			Extracting physical	information from data					
Ihe Wea	ak Adversarial Netv	work (VVAN)	Simplifying complex	x processes					

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MUSCL-THINC-BVD

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Consider the invisid single- and two-phase compressible flows:

$$rac{\partial \mathbf{U}}{\partial t} +
abla \cdot \mathbf{F} = \mathbf{S}.$$

• Inviscid Euler equation:

$$oldsymbol{U} = \left(egin{array}{c}
ho \
ho oldsymbol{V} \ E \end{array}
ight), \quad oldsymbol{F}(oldsymbol{U}) = \left(egin{array}{c}
ho oldsymbol{V} \otimes oldsymbol{V} + poldsymbol{I} \ (E+p)oldsymbol{V} \end{array}
ight), \quad oldsymbol{S} = \left(egin{array}{c} 0 \ 0 \ 0 \ 0 \end{array}
ight).$$

• The five-equation model for two-phase inviscid compressible flow:

$$oldsymbol{U} = egin{pmatrix}
ho_1 lpha_1 \
ho_2 lpha_2 \
ho oldsymbol{V} \
ho oldsymbol{V} \
ho oldsymbol{V} \otimes oldsymbol{V} + poldsymbol{I} \
ho oldsymbol{V} \otimes oldsymbol{V} + poldsymbol{I} \ (E+p)oldsymbol{V} \ lpha_1oldsymbol{V} \end{pmatrix}, oldsymbol{S} = egin{pmatrix} 0 \ 0 \ 0 \ 0 \ lpha_1 \
alpha_1 \
alpha_1 \end{pmatrix} egin{pmatrix} oldsymbol{F}(oldsymbol{U}) = egin{pmatrix}
ho_1 lpha_1 lpha_1 oldsymbol{V} \ lpha_1 oldsymbol{V} \end{pmatrix}, oldsymbol{S} = egin{pmatrix} 0 \ 0 \ 0 \ lpha_1 \
alpha_1 \
alpha_1 \
alpha_1 \
alpha_1 oldsymbol{V} \end{pmatrix} egin{pmatrix} oldsymbol{F}(oldsymbol{U}) = egin{pmatrix}
ho_1 lpha_1 lpha_1 oldsymbol{V} \
alpha_1 oldsymbol{V} \
alpha_1 oldsymbol{V} \
alpha_1 oldsymbol{V} \end{pmatrix}, oldsymbol{S} = egin{pmatrix} oldsymbol{O} \ \holdsymbol{O} \
alpha_1 oldsymbol{V} \
alpha_2 ol$$

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For close the system, we apply the stiffened equation of state (EOS):

• Inviscid Euler equation:

$$p(
ho,e)=(\gamma-1)
ho\left(e-e_{\infty}
ight)-\gamma p_{\infty},$$

• The five-equation model for two-phase inviscid compressible flow:

$$p_k(
ho_k,e_k)=(\gamma_k-1)
ho_k\left(e_k-e_{k,\infty}
ight)-\gamma p_{k,\infty},$$

we need to determin an additional EOS of the mixture:

$$p\left(
ho,e,lpha_1,lpha_2
ight)=\left(
ho e-\sum_{k=1}^2lpha_k
ho_k e_{\infty,k}+\sum_{k=1}^2rac{lpha_kp_{\infty,k}}{\gamma_k-1}
ight)/\sum_{k=1}^2rac{lpha_k}{\gamma_k-1},$$

where

 $ho e=lpha_1
ho_1e_1+lpha_2
ho_2e_2.$

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Admissible reconstruction scheme

The MUSCL scheme

$$Q_{i}^{M}(x) = ar{u}_{i} + \sigma_{i}rac{x-x_{i}}{\Delta x_{i}}, \hspace{1em} x \in [x_{i-1/2}, x_{i+1/2}],$$

where \bar{u}_i is the cell-average value, and σ_i is the slope defined in cell C_i , accompied by proper slope limiter $\tilde{\Delta}$:

$$\sigma_i = ilde{\Delta}(ar{u}_i - ar{u}_{i-1}, ar{u}_{i+1} - ar{u}_i).$$

The THINC scheme

$$Q_i(x)_i^T = ar{u}_{\min} + rac{ar{u}_{\max} - ar{u}_{\min}}{2} \left(1 + heta anh \left(eta rac{x - x_{i-1/2}}{x_{i+1/2} - x_{i-1/2}} - ilde{x}_i
ight)
ight), \quad x \in [x_{i-1/2}, x_{i+1/2}],$$

where $\bar{u}_{\min} = \min(\bar{u}_{i-1}, \bar{u}_{i+1}), \bar{u}_{\max} = \max(\bar{u}_{i-1}, \bar{u}_{i+1})$ and $\theta = sgn(\bar{u}_{i+1} - \bar{u}_{i-1})$. β is selected to control the thickness of the jump, which is always valued from 1.4 to 2.0.

Assume that left- and right-side values of an arbitrary cell boundary I_i are U^L and U^R , the approximate Rimenan solver in the canonical formulation is given:

$$F(U^L, U^R) = \frac{1}{2} \underbrace{(F(U^L) + F(U^R))}_{\text{Central part}} - \underbrace{A(U^L, U^R)(U^R - U^L)}_{\text{Dissipative part}}.$$

Here, $F(U^L, U^R)$ denotes the numerical flux between the I_i and $A(U^L, U^R)$ is the matrix computed from U^L, U^R .

The BVD scheme is designed to select the reconstruction function from multiple candidates so as to minimize the dissipation term.

$$\Xi:=\left\{Q_i^{\xi_1}(x),Q_i^{\xi_2}(x),\cdots,Q_i^{\xi_N}(x)
ight\} {\longrightarrow \atop \longrightarrow} Q_i^{\xi_{opt}}(x), \quad ext{on } C_i.$$

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	$\hat{u}_i(x)^{BVD} = egin{cases} & \ & \ & \ & \ & \ & \ & \ & \ & \ & $	$egin{aligned} \hat{u}_i(x)^{THINC}, ext{ if } \delta < & \ ext{and } TI \ \hat{u}_i(x)^{MUSCL}, ext{otherw} \ \{f(M,M,s),f(T,T), T_{i-1/2}^{R,s} ight + igg \hat{u}_{i+1/2}^{L,s} \ + igg \hat{u}_{i+1/2}^{L,s} \end{aligned}$	$C < 1-\delta, ext{ and } (ar{u}_{i+1}) \ BV^{THINC}_{i,\min} < TBV^{MUS}_{i,\min}, \ T,s), f(M,T,s), f(T,s), $	$egin{aligned} &-ar{u}_i)(ar{u}_i-ar{u}_{i-1})>0,\ &CL,\ &M,s)\},\ &\in \{MUSCL,THINC\}. \end{aligned}$	
	C_{i-1}	$u_{i-\frac{1}{2},L}^{MUSCL}$	C_{i} $u_{i+\frac{1}{2},L}^{THINC}$ $u_{i+\frac{1}{2},R}^{THINC}$ $u_{i+\frac{1}{2},R}^{THINC}$ $x_{i+\frac{1}{2}}$	C_{i+1} $\hat{u}_{i+1}(x)^{THINC}$ C $x_{i+\frac{3}{2}}$	
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The BVD scheme is summarized as following steps:

<pre>// 1. Reconstruct all admissible reconstructions</pre>	1	<pre>// 1. Reconstruct all admissible reconstructions</pre>
for cell in all cells:	2	for cell in all cells:
for scheme in admissible set:	3	for scheme in admissible set:
reconstruct and save scheme in the cell	4	reconstruct and save scheme in the cell
	5	
<pre>// 2. Select optimal canditate reconstruction</pre>	6	<pre>// 2. Select optimal canditate reconstruction</pre>
for cell in all cells:	7	for cell in all cells:
for scheme in candidate set:	8	for scheme in candidate set:
find minimal boundary variation of all schemes	9	find minimal boundary variation of all schemes
	10	
<pre>// 3. Calculate the cell boundary value</pre>	11	<pre>// 3. Calculate the cell boundary value</pre>
for cell in all cells:	12	for cell in all cells:
using selected scheme from (2) to reconstruct	13	using selected scheme from (2) to reconstruct

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Deep MUS	CL-THINC-E	SVD			

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	BVD					

- MUSCL-THINC-BVD
 - Reconstruct all candidate schemes
 - Calculate the total boundary variation (TBV)
 - Select the scheme leading to minimum TBV

Pros:

- reduce the dissipative error
- Directly extend to the multi candidate reconstructions

Cons:

- Every cell needs to reconstruct all possible scheme
- Encountered with multi candidate reconstructions is cost expensively

DeepMTBVD

- Select the scheme by Neural network
- Reconstruct the selected scheme in each cell

Pros:

- Reduce the dissipative error same to MUSCL-THINC-BVD
- Extend to the multi-candidates without expensive cost

ntroduction 000000	MUSCL-THINC-B 0000000	VD Dee OO	pMTBVD ●OOO	Numer 0000	ical Method O	Numerical results 0000000000000000	Conclusion 000
		C_{i-2}	C_{i-1}	C_i	C_{i+1}	C_{i+2}	
	ŀ			stencil _i			
			stencil _{i-1}	i< →	$stencil_{i+1}$	> i	

Observation:

1. The stencils of BVD scheme consists of $\{C_{i-2}, C_{i-1}, C_i, C_{i+1}, C_{i+2}\}$.

The BVD scheme describes the discontinuity which the relative value among the neighboring cells.
 2.

$$rac{ar{u}^s-ar{u}_{ ext{min}}}{ar{u}_{ ext{max}}-ar{u}_{ ext{min}}}\in [0,1], \quad s\in \{ ext{MUSCL, THINC}\}.$$

Here, $ar{u}_{\min} = \min(ar{u}_{i-1}, ar{u}_{i+1}), ar{u}_{\max} = \max(ar{u}_{i-1}, ar{u}_{i+1}).$

Normalization formualtion:

$$ilde{u}_i = \left\{egin{array}{c} rac{ar{u}_i - ar{u}_{\min}}{ar{u}_{\max} - ar{u}_{\min}}, & ar{u}_{\max}
eq ar{u}_{\min}, \ 0, & ar{u}_{\max} = ar{u}_{\min} < 10^{-15}, \ 1, & ar{u}_{\max} = ar{u}_{\min} > 10^{-15}. \end{array}
ight.$$

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Sturcture o	f the Neural	Network			

The DeepMTBVD $\mathcal{N}(\tilde{u}_{i-2}, \tilde{u}_{i-1}, \tilde{u}_i, \tilde{u}_{i+1}, \tilde{u}_{i+2}, \Delta x; \Theta)$ with two hidden layers is described as following:



In the figure abovce the output p is probability of the THINC scheme.

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Loss function

Let the label and probability $\left(p_{y}
ight)$ of the THINC scheme and MUSCL scheme

$$p_y = \left\{egin{array}{ccc} p, & y=1, \ 1-p, & y=0. \end{array}
ight. egin{array}{ccc} y = \left\{egin{array}{ccc} 1, & ext{THINC} \ 0, & ext{MUSCL} \end{array}
ight.$$

Let loss function is reformulated as:

$$loss = \sum_{i=1}^N FL(p_y^i),
onumber \ FL(p_y) = -\omega_t (1-p_y)^\gamma log(p_y).$$

Remark:

- Loss learn the features from the imbalanced data.
- The weight ω_0 for MUSCL is larger than ω_1 for THINC.

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Data gar	oration	00000		000000000000000000000000000000000000000	000	

- 1. Set different mesh size Δx .
- 2. Randomly set up the initial data at x = 0.5 in a sod tube.

$$(
ho_i^{(0)}, u_i^{(0)}, p_i^{(0)}) = egin{cases} (
ho_i^L, u_i^L, p_i^L), & x \leq 0.5, \ (
ho_i^R, u_i^R, p_i^R), & x \geq 0.5. \end{cases}$$

- 3. Calculate several time steps (about 10) by third-order SSP Runge-Kutta.
- 4. Given target cell C_i , we mark it as "1" if it is reconstructed by THINC, otherwise it is marked as "0". we record the $(\bar{u}_{i-2}, \bar{u}_{i-1}, \bar{u}_i, \bar{u}_{i+1}, \bar{u}_{i+2}, \Delta x, y)$ as one sample.
- 5. Change the mesh size and initial data and proceed to the next round.

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Dividing the computational domain into N cells, and denotes i^{th} cell $C_i = [x_{i-1/2}, x_{i+1/2}], i = 1, \dots, N$. Assume that grid size is $\Delta x_i = x_{i+1/2} - x_{i-1/2}$ and cell center locates at $x_{i+1/2}$, the general integral formulation of conservative variable \mathbf{U} over a finite volume cell C_i , which follows the cell-average values:

$$\overline{oldsymbol{U}}_i = rac{1}{\Delta x_i}\int_{x_{i-1/2}}^{x_{i+1/2}}oldsymbol{U} dx.$$

The Euler equation or five-equation model is reformulated as:

$$rac{d}{dt}\overline{oldsymbol{U}}_i = -rac{1}{\Delta x_i} \left(\underbrace{\mathcal{A}^+\Deltaoldsymbol{U}_{i-1/2}}_{ ext{right-moving fluctuations}} + \underbrace{\mathcal{A}^-\Deltaoldsymbol{U}_{i+1/2}}_{ ext{left-moving fluctuations}} + \underbrace{\mathcal{A}\Deltaoldsymbol{U}_i}_{ ext{total fluctuation}}
ight),$$

where $\mathcal{A}^+ \Delta oldsymbol{U}_{i-1/2}, \mathcal{A}^- \Delta oldsymbol{U}_{i+1/2}, \mathcal{A} \Delta oldsymbol{U}_i$ is calculated by wave-propgation method.

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$$egin{aligned} \mathcal{A}^{\pm}\Delta oldsymbol{U}_{i\mp1/2} &= \sum_{k=1}^{K} \left[s^k \left(oldsymbol{U}_{i-1/2}^{L},oldsymbol{U}_{i+1/2}^{R}
ight)
ight]^{\pm} oldsymbol{W}^k \left(oldsymbol{U}_{i-1/2}^{L},oldsymbol{U}_{i+1/2}^{R}
ight), \ \mathcal{A}\Delta oldsymbol{U}_{i} &= \sum_{k=1}^{K} \left[s^k \left(oldsymbol{U}_{i-1/2}^{R},oldsymbol{U}_{i+1/2}^{L}
ight)
ight]^{\pm} oldsymbol{W}^k \left(oldsymbol{U}_{i-1/2}^{R},oldsymbol{U}_{i+1/2}^{L}
ight), \end{aligned}$$

where K, s^k and W^k are the number of waves, moving speeds, and jumps of three propagation discontinuities solved by Riemann solvers such as HLL/HLLC.

Once given spatial discretization, we apply the two-stage second-order SSP Runge-Kutta for temporal evolution

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$$egin{aligned} \overline{oldsymbol{U}}^* &= \overline{oldsymbol{U}}^n + \Delta t \mathcal{L}\left(\overline{oldsymbol{U}}^n
ight), \ \overline{oldsymbol{U}}^{n+1} &= rac{1}{2}\overline{oldsymbol{U}}^n + rac{1}{2}\left(\overline{oldsymbol{U}}^* + \Delta t \mathcal{L}\left(\overline{oldsymbol{U}}^*
ight)
ight). \end{aligned}$$

DeepMTBVD

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Alç	gorithm of DeepM	FBVD					
	 // Select input stencil for each for i in all cells: stencil[i][0] = u[i-2] stencil[i][1] = u[i-1] stencil[i][2] = u[i] stencil[i][3] = u[i+1] stencil[i][4] = u[i+2] stencil[i][5] = dx // Inferrenced neural network prediction = network→inferrence // Recontructed by prediction for i in all cells: if prediction[i] < p0: cell i is reconstructed by M else cell i is reconstructed by T // Advance to next sub-time stem 	:h cell ?(stencil) MUSCL FHINC ?p		We perform in rather than one inferrencing or efficient as infe p_0 is a parame Generally we ve specified in the Conservative of quantities can framework.	ference on all cells a e cell at a time. In pr ne by one is not as fa errencing together. ter, which is depend will take 0.5, unless e numerical example juantities and charad also be trained unde	at once, ractice, ast and on problem otherwise e. cteristic er this	1.
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Sod's Problem						
$(ho_0, v_0, p_0) = igg\{$	$egin{array}{llllllllllllllllllllllllllllllllllll$	$0\leqslant x\leqslant 0.5 \ ext{otherwise}$.	Pressure	Exact MUSCL MUSCL-THINC-BVD o deepMTBVD	Exact Exact 0.8 - • MUSCL 0.6 - • •	city B
Time(s)	MUSCL-THINC-BVD	DeepMTBVD	0.4 -		0.4 -	
Reconstruction Time	48.6	11.8	0.2 -		0.2 -	2
Selection Time	4.4	36.2	0.0 0.2 0.4 0.6	0.8 1.0	0.0 - 0.2 0.4	0.6 0.8 1.0
Total Time	53.0	48.0	1.0 - Density	Exact	0.425 - 0.425 - 0.425	rspective
■ N = 100000			08.	MUSCL-THINC-BVD deepMTBVD	0.400 -	
Time(s)	MUSCL-THINC-BVD	DeepMTBVD	0.6		0.375 -	
Reconstruction Time	4951.3	1139.1			0.325 -	a Ə
Selection Time	650.1	3734.6	0.7		0.300 -	0
Total Time	5601.4	4873.7	0.0 0.2 0.4 0.6	0.8 1.0	0.66 0.68 0.70 0.72	0.74 0.76 0.78 0.80
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ntrodu DOOC	ction DOO	MUSCL-THINC-BVD 0000000	DeepMTBVD 000000	Numerical Method	Numerical resultsConclOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOO	usion)
Bla	ast wave					
	(ρ, u, p) • N = 10000	$=egin{cases} (1.0, 0.0, 1000.0, \ (1.0, 0.0, 0.01) \end{pmatrix}$	$) { m if} \ x < 0.5 \ { m otherwise}$	6- 5- 8- 3-	Exact MUSCL deepMTBVD	
	Time(s)	MUSCL-THINC-BVD	DeepMTBVD	2 -	0.5 0.66 0.68 0.70 0.72 V.	
	Reconstruction Time	60.0	12.6	1-		
	Selection Time	5.1	42.2		0.0 0.2 0.4 X 0.6 0.8	1.0
	Total Time	65.1	54.8	6 -	Exact MUSCL-THINC-BVD deepMTBVD	
I	N = 100000			1-25	12 - 0	
	Time(s)	MUSCL-THINC-BVD	DeepMTBVD	4-	1.1 - 1.0 - 0.9 -	
	Reconstruction Time	4746.5	782.9	2 -		
	Selection Time	600.8	2655.2	1-	0.00 0.08 0.70 0.72	
	Total Time	5347.3	3438.1	l	0.0 0.2 0.4 X 0.6 0.8	1.0
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Simple i	nterface probl	em				
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Simple interface problem

if x < 0.4 or x > 0.6:

$$(lpha_1,
ho,u,p)=(10^{-8},1,1000,100000),$$

 $\text{if } 0.4 \leq x \leq 0.6.$

$$(lpha_1,
ho,u,p)=(1-10^{-8},1000,1000,100000).$$

The parameters of stiffened EOS are : $\gamma_1=4.4, p_{\infty,1}=6 \times 10^8, e_{\infty,1}=0\gamma_2=1.4, p_{\infty,2}=0, e_{\infty,2}=0.$ In this example, p_0 is set to be 0.4.



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DeepMTBVD

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Underwate	Underwater explosion									

The computation domain is $[-2m, 2m] \times [-1.5m, 2.5m]$. The diameter of cylindrical air cavity is 0.24m which initially centered at (0, -0.3)m with high pressure 10^9 Pa and high density 1250kg/m^3 . The planar water-air interface is located at y = 0m. Both water and air mediums are under standard atmospheric conditions and stay stationary initially. Under this assumption, The density of pure water is 1000kg/m^3 and pure air is 1kg/m^3 . In water medium, the volume fraction of air $\alpha_1 = 10^{-8}$ and vice versa in air.







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			T=0.	20ms	
		-			
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Compare with the result from Shukla $^{[1]}$ among x=0 at t=0.2ms

Compare with the result from $\operatorname{Hu}^{[2]}_{=2}$ among x=0 at t=0.2ms



- 1. R. K. Shukla, C. Pantano, and J. B. Freund. An interface capturing method for the simulation of multi-phase compressible flows. Journal of Computational Physics, 229(19):7411–7439, 2010. 🔁
- 2. X. Y. Hu, N. A. Adams, and G. Iaccarino. On the hllc riemann solver for interface interaction in compressible multi-fluid flow. Journal of Computational Physics, 228(17):6572–6589, 2009. 🔁

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ntroduction	MUSCL-THINC-BVD	DeepMTBVD	Numerical Method	Numerical results	Conclusion					
DOOOOO	0000000	000000		0000000000000000	000					
Two-dimen	Two-dimensional shock–R22-cylinder interaction									

In the R22 bubble,

$$(
ho_1,
ho_2,u_1,u_2,p,lpha_1)=\left(3.869~{
m kg/m}^3,1.225~{
m kg/m}^3,0.0,0.0,1.01325 imes 10^5~{
m Pa},1.0-10^{-8}
ight),$$

while outside the bubble, the parameters in the pre-shock are:

$$(
ho_1,
ho_2,u_1,u_2,p,lpha_1)=\left(3.869~{
m kg/m}^3,1.225~{
m kg/m}^3,0.0,0.0,1.01325 imes 10^5~{
m Pa},10^{-8}
ight),$$

and the corresponding parameters in the post-shock are:

$$(
ho_1,
ho_2,u_1,u_2,p,lpha_1)=ig(3.869~{
m kg/m}^3,1.686~{
m kg/m}^3,-113.5~{
m m/s},0.0,1.59 imes10^5~{
m Pa},10^{-8}ig)$$



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Two-din	nensional sho	ck-helium-	bubble intera	action		
	(10^{-1})	$^{-8}, 0.167~{ m kg/m^3}, 0.0$	$(0, 0.0, 1.01325 imes 10^5 ~{ m Pa})$	Helium bubb	le,	

$$(lpha_1,
ho,u,v,p) = egin{cases} (10^{-1},0.101\,{
m kg/m}\,,0.0,0.0,1.01325 imes10^{-1}\,{
m a}) & {
m inclum busines} \ (1.0-10^{-8},1.686\,{
m kg/m}^3,113.5\,{
m m/s},0.0,1.59 imes10^{5}\,{
m Pa}) & {
m Post-shock}\,, \ (1.0-10^{-8},1.225\,{
m kg/m}^3,0.0,0.0,1.01325 imes10^{5}\,{
m Pa}) & {
m Otherwise}. \end{cases}$$

The corresponding parameters of EOS are $\gamma_1=1.4, p_{\infty,1}=e_{\infty,1}=0$ and $\gamma_2=1.667, p_{\infty,2}=e_{\infty,2}=0$



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Introduction 000000	MUSCL-THINC-BVD	DeepMTBVD 000000	Numerical Method	Numerical results	Conclusion ●○○		
Numerical Results							

- Introduction
- MUSCL-THINC-BVD
- Deep-MUSCL-THINC-BVD(DeepMTBVD)
- Numerical Method
- Numerical Result
- Conclusion

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ntroduction	MUSCL-THINC-BVD	DeepMTBVD	Numerical Method	Numerical results	Conclusion	
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Conclu	ision					

- A novel low-dissipative scheme is proposed, DeepMTBVD, which is based on BVD scheme and the artificial neural network.
- The DeepMTBVD is trained from the data obtained by 1D Euler equation, and then is successfully extended to multi-dimensional and two-phase flow.
- Compared with MUSCL scheme, the DeepMTBVD shows the lower dissipation, while compared to MTBVD, the DeepMTBVD achieves comparable results with less computational time.
- In the future, we will extend this method into multi-candidate reconstructions and into unstructured grid.

Thank you for listening!

Minsheng Huang

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