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[13-B-03] Grey area mitigation in Equivalent-DES using commutation error estimators

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Grey area mitigation in Equivalent-DES using commutation error estimators

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Abstract: Over the past few decades, hybrid RANS/LES methods for modeling turbulence have become a popular compromise between the computationally cheap Reynolds Averaged Navier-Stokes (RANS) approach, and the reliable Large Eddy Simulation (LES). But they still face conceptual issues when transitioning from RANS to LES (or reversely), such as log-layer mismatch, modeled scale depletion, globally labeled as grey area issues. From a theoretical point of view, this issues may be related to the commuation error arising when the hybrid RANS/LES filter varies rapidly. Following the additive filter method, initially presented by Germano (*Theoretical and Computational Fluid Dynamics, 17(4):225–231, 2004*), we present a method aimed at estimating the aforementioned commutation errors.

Keywords: turbulence modeling, hybrid RANS/LES, greay area mitigation

1 Introduction

In seamless hybrid RANS/LES (sHRL) methods like the family of DES models, the filtering operator is a complex blending between the ensemble-average (associated with RANS) and convolution filtering (associated with LES) of the Navier-Stokes equation. The filtered NS equations are interpreted as a system of equations for filtered flow variables. Unfortunately, differential operators do not commute with the sHRL filtering operation: if the filter varies in space and/or in time, the derivative of a filtered quantity is not equal to the filtering of the derivative of that quantity. Formally, a commutation error E_c then arises:

$$E_{ci} \equiv \widetilde{\frac{Du_i}{Dt}}^H - \frac{D\widetilde{u}_i^H}{Dt}. \tag{1}$$

where the operator $\widetilde{.}^H$ is sHRL filtering. This error is non-negligible in areas with steep transition in resolution level (see e.g. [1]). Although it may not be the only source, this commutation error is undeniably one cause of the modeled scale depletion / log-layer mismatch issues which plague sHRL methods. A simple and comprehensive correction to this issue is presented hereafter.

2 Flow equations

The sHRL flow equations used are stated here for a constant filter. Commutation errors appear when the filter is spatially non-constant, which is the case for the hybrid filter $\widetilde{.}^H$. These errors are analysed in the next section.

Filtered Navier-Stokes equations Seamless HRL uses a blend of RANS and LES which are both based on the filtered incompressible Navier-Stokes equations:

$$\frac{\partial \widetilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widetilde{u}_i \widetilde{u}_j\right) = -\frac{1}{\rho}\frac{\partial \widetilde{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(2\nu \widetilde{S}_{ij} - \widetilde{u"_i u"_j}\right), \tag{2}$$

$$\frac{\partial \widetilde{u}_i}{\partial x_i} = 0. \tag{3}$$

These are obtained by introducing a decomposition $u = \widetilde{u} + u"$ and $p = \widetilde{p} + p"$, where $\widetilde{u}$ and $\widetilde{p}$ are the filtered velocity field and pressure field respectively. The quantity $\tau_{ij} = -\widetilde{u"_i u"_j}$ is the subfilter-stress tensor which is given by the turbulence model, ρ the density of the flow, ν the molecular kinematic viscosity and S_{ij} the filtered strain-rate tensor $\widetilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \widetilde{u}_i}{\partial x_j} + \frac{\partial \widetilde{u}_j}{\partial x_i}\right)$.

Turbulence model The chosen turbulent closure is $k - \omega$ SST [2], for which the equation of the unresolved turbulent kinetic energy k_u reads:

$$\frac{\partial k_u}{\partial t} + \frac{\partial}{\partial x_j}\left(\widetilde{u}_j k_u - (\nu + \sigma_k \nu_t)\frac{\partial k_u}{\partial x_j}\right) = \tau_{ij}\frac{\partial \widetilde{u}_i}{\partial x_j} - \beta^* \omega^* k_u. \tag{4}$$

The hybridization is performed on the sink term of this equation, following the "Equivalent DES" method described in [3], which bases the blending on the energy ratio parameter r_k, defined as the fraction of modeled turbulent kinetic energy k_u to total (modeled+resolved) turbulent kinetic energy k_{tot}.

$$\underbrace{\beta^* \omega^* k_u}_{\text{RANS}} \rightarrow \underbrace{\frac{k_u^{3/2}}{L}}_{\text{hybrid}} \quad \text{with} \quad L = \frac{r_k^{3/2}\sqrt{k_{\text{tot}}}}{C_\mu \psi \omega_{\text{tot}}}, \tag{5}$$

where ω^* is the transported specific dissipation (second scale of the turbulence model), and ψ is defined as [4]:

$$\psi = \frac{\beta}{C_\mu \gamma + r_k\left(\beta - C_\mu \gamma\right)} \tag{6}$$

with the parameters β, C_μ and γ defined as in the RANS model [2]. The target energy ratio is:

$$r_k = \frac{1}{\beta_0}\left(\frac{\pi\sqrt{k_{\text{tot}}}}{\Delta C_\mu \omega_{\text{tot}}}\right)^{-2/3}, \quad \beta_0 = 0.2, \quad \Delta = \Omega_{\text{cell}}^{1/3}. \tag{7}$$

Finally, the subgrid stress is approximated as $\tau_{ij} = 2\nu_u \widetilde{S}_{ij}$, where the eddy viscosity related to the unresolved motion is given by [4]:

$$\nu_u = \frac{a_1 k_u}{\max\left[a_1 \psi \omega^*; \widetilde{S} F_2\right]}, \tag{8}$$

where $\widetilde{S} = \sqrt{2\widetilde{S}_{ij}\widetilde{S}_{ij}}$ and F_2 is formally identical to its RANS version.

3 Estimation of the commutation error

Following the additive filter idea of Germano [5], a class of hybrid sHRL filters H may be expressed as:

$$\widetilde{\cdot}^H = b\,\widetilde{\cdot}^F + (1-b)\,\widetilde{\cdot}^E \tag{9}$$

where $\widetilde{\cdot}^F$ represents a filter which is arbitrarily well-resolved and $\widetilde{\cdot}^E$ the ensemble-average. The parameter b is a blending factor, which may vary in space and time. The above equation allows to express the commutation error E_c between the H-filtering and any differential operator $\mathcal{D}_i$:

$$\widetilde{\mathcal{D}_i \cdot}^H = \mathcal{D}_i\,\widetilde{\cdot}^H - \frac{\partial b}{\partial x_i}\left(\widetilde{\cdot}^F - \widetilde{\cdot}^E\right) \tag{10}$$

In the present work, an estimation of the commutation error E_c is presented, by (i) linking the blending parameter b in Eq. (9) to the energy ratio parameter r_k defined in section 2, and (ii) choosing $F \equiv I$, i.e. no filtering at all.

Blending parameter To establish a relationship between the blending factor b and the energy ratio r_k, we use the expression of the turbulent stress at the H level, since the cutoff only appears explicitly in the turbulence equations. Recalling the turbulent stresses $\tau_{ij}^{\mathcal{O}}$ associated with an arbitrary filter $\mathcal{O}$, i.e. the part of the velocity fluctuations filtered by $\mathcal{O}$:

$$\tau_{ij}^{\mathcal{O}} = \widetilde{u_i u_j}^{\mathcal{O}} - \widetilde{u_i}^{\mathcal{O}} \widetilde{u_j}^{\mathcal{O}} , \tag{11}$$

and substituting the definition of the blended filter Eq. (9), one may express the ensemble-averaged turbulent stress at the H level, keeping in mind that $\tau_{ij}^{F} = 0$:

$$\widetilde{\tau_{ij}^{H}}^{E} = \left(1 - b^2\right) \tau_{ij}^{E} . \tag{12}$$

Keeping in mind that $\widetilde{\tau_{ii}^{H}}^{E} = r_k \tau_{ii}^{E}$ and assuming isotropy of b and r_k, one establishes a relationship between b and r_k:

$$b = \sqrt{1 - r_k}. \tag{13}$$

Momentum equations With b known, Eq. (10) can now be applied to the flow equations, to estimate commutation correction terms. For the momentum equations, assuming b constant in time, the correction E_{ci} reads:

$$\begin{aligned} E_{ci} = & \frac{1}{\rho}\frac{\partial b}{\partial x_i}\left(\widetilde{p}^F - \widetilde{p}^E\right) + \frac{\partial b}{\partial x_j}\left(\widetilde{u_i}^F \widetilde{u_j}^F - \widetilde{u_i}^E \widetilde{u_j}^E\right) \\ & - \frac{\partial b}{\partial x_j}\left(\tau_{ij}^E\right) - 2\nu \frac{\partial b}{\partial x_j}\frac{\partial}{\partial x_j}\left(\widetilde{u_i}^F - \widetilde{u_i}^E\right) - \nu \frac{\partial^2 b}{\partial x_j^2}\left(\widetilde{u_i}^F - \widetilde{u_i}^E\right) . \end{aligned} \tag{14}$$

Continuity equation Regarding the continuity equation:

$$E_{c,\mathrm{cont}} = \widetilde{\frac{\partial u_j}{\partial x_j}}^{H} - \frac{\partial \widetilde{u}_j^H}{\partial x_j} = \frac{\partial b}{\partial x_j}\left(\widetilde{u_j}^F - \widetilde{u_j}^E\right). \tag{15}$$

While this correction implies that the divergence of the H-filtered velocity field is no longer zero, except in average, it might remain small (c.f. [1]).

Turbulence model The total turbulent kinetic energy (resolved+modeled) must remain unaffected by extra terms accounting for commutation errors. Therefore, as E_c changes the balance of resolved turbulent kinetic energy, this must be compensated at the subfilter level. As discussed in [6], the exact derivation of the commutation error terms is cumbersome. Instead, the aforementioned paper, as well as Hamba [7], add the following term in the equation for the subfilter turbulent kinetic energy:

$$E_{ck} = E_{ci}\left(\widetilde{u}_i^E - \widetilde{u}_i^H\right), \tag{16}$$

which is convenient since E_{ci} is computed anyway. For the turbulent length scale ω, it is assumed that:

$$E_{c_\omega} = \frac{\omega}{k_u} E_{c_k}. \tag{17}$$

4 Reconstructing fluctuations

The E- and F-filtered quantities in (14) to (17) are unknown, since only the H-filtered solution is computed. The E-filtered solution can be easily obtained by ensemble averaging the H-solution, but as $\widetilde{\cdot}^F = I$, the F-filtered solution corresponds to a DNS result and should resolve the turbulence on all scales. This resolved turbulence has to be reconstructed.

In [1], the F-fields are extrapolated from the difference between E and H. Thus, the F-filtering operation over any quantity ϕ yields (as long as $b \neq 0$):

$$\widetilde{\phi}^F = \widetilde{\phi}^H + \frac{1-b}{b}\left(\widetilde{\phi}^H - \widetilde{\phi}^E\right) \;\Leftrightarrow\; \widetilde{\phi}^F = \widetilde{\phi}^E + \frac{\widetilde{\phi}^H - \widetilde{\phi}^E}{b}. \tag{18}$$

In actual simulations on finite-sized meshes, the finest turbulent scales cannot be resolved. Therefore, (18) implies that the commutation errors are corrected by adding or removing resolved turbulence only in the larger scales. Similar to [8], it is assumed that natural cascading will lead to the creation of smaller scales, if these can be resolved locally.

Furthermore, the above formulation is invalid for the case $b = 0$, which corresponds to the RANS limit. Within the RANS area, this is no problem, since E_c vanishes anyway. But at the limit between $b = 0$ and $b > 0$, Eq. (18) cannot provide the F-filtered variables anymore. In this case, there are two possibilities:

1. preventing $b = 0$, by setting a minimum value b_{min}. In [1], $b_{min} = 0.15 \Leftrightarrow r_{k,max} \approx 0.98$.
2. reconstructing the F-field from the E-field and the τ^E_{ij} stresses, instead of from the E- and H-fields. A similar strategy has been successfully used by [9].

The second solution above is implemented as:

$$\widetilde{u}_i^F = \widetilde{u}_i^H + \underbrace{\sqrt{\tau^H_{ii}}\mathcal{N}(0,1)}_{\approx\, u_i''^H} \ , \quad \widetilde{p}^F = \widetilde{p}^H + \underbrace{\frac{\rho\tau^H_{ii}}{2}\mathcal{N}(0,1)}_{\approx\, p''^H} \ , \tag{19}$$

where $\mathcal{N}(0,1)$ is a random number with a normal distribution of average 0 and standard deviation 1. Regarding the F velocities in Eq. (19), the Einstein convention does not apply.

5 Numerical methodology

The computations were run using *Code_Saturne*. This open-source CFD code developed by the French electricity producer E.D.F. allows the resolution of the equations of Navier-Stokes in 2D or 3D, and includes RANS and LES models for the study of turbulent flows. Except DDES, hybrid RANS/LES models are not supported in the standard version of the solver, but have been implemented by the first author.

Code_Saturne uses a cell-centred finite-volume approach that handles unstructured meshes with arbitrary cell types, although the current paper uses structured full-hexahedral grids. Pressure-velocity coupling is achieved with a SIMPLEC-type splitting method and Rhie & Chow interpolation for the face fluxes, to prevent odd-even decoupling. A range of convective schemes is available; for maximum precision in solving turbulent flows, and in order to encourage fluctuations, a fully centred scheme is used. A Crank-Nicholson scheme is used for the time integration.

6 Test case and results

The commutation error correction is tested on a periodic turbulent channel flow, which is the most suitable test case for non-streamwise RANS/LES transition. The friction Reynolds number Re_τ is 395. Computations were performed on a $40 \times 54 \times 40$ grid, with stretching in the y (cross-channel) direction and uniform cell sizes in the other directions. The boundary conditions are a periodic condition on the inflow and outflow faces, no-slip on the channel walls, and periodicity in the z-direction. The flow is initialized with resolved and modelled turbulence, and then simulated over 300 flow-through times. In order to avoid bias due to discrepancies between *observed* r_k and *target* r_k, a specific forcing, not detailed here, is used to reduce these discrepancies.

To assess the effect of accounting for commutation errors, three configurations will be considered:

(A) Basic Equivalent-DES, initialized with the proper ratio between modeled and resolved turbulent kinetic energy,

(B) Same as (A) + commutation error using Eq. (18) to estimate the F-fields.

(C) Same as (A) + commutation error using Eq. (19) to estimate the F-fields.

In all three cases, the energy partition is enforced during the computation, with a volume forcing.

It is worth mentioning that the computation of the full model was instable and diverged. Therefore, cases B an C consider only E_c as prescribed by Eq. 14.

Figure 1 compares the mean streamwise velocity in all three cases with DNS and $k-\omega$ SST RANS. The flow rate is globally overestimated by all three hybrid approaches, which is classic in sHRL, but supposed to be circumvented, at least partially, by grey area mitigation. Instead, the overprediction, by cases B and C, of the streamwise velocity in the channel centre, is even worse than case A. Knowing that

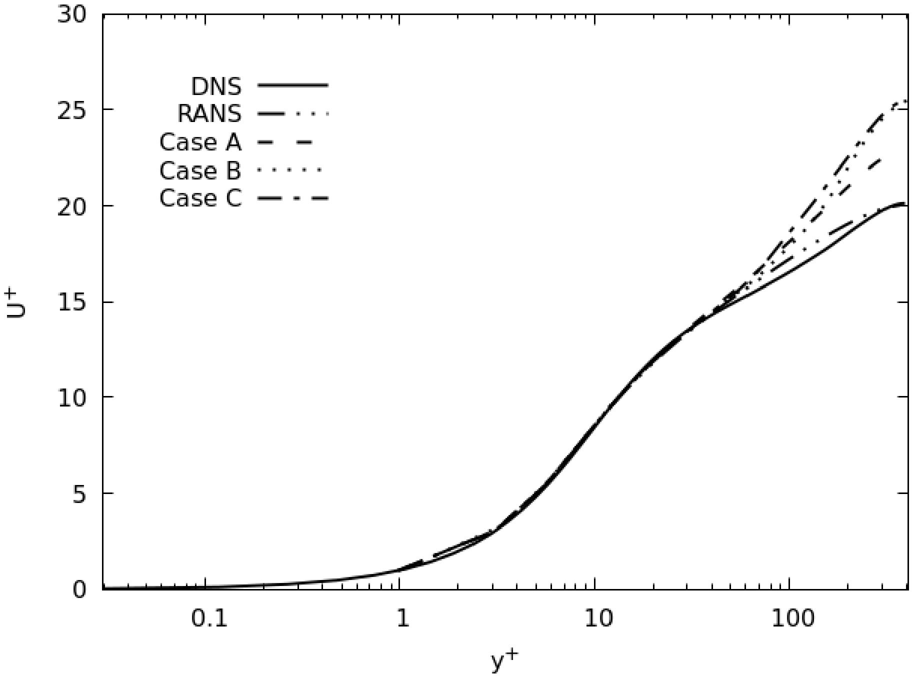

Figure 1: Mean streamwise velocity

in [1], the velocity is properly predicted by blending an averaged solution and a filtered one, the most obvious possibility to improve the current strategy is to reconsider the way of estimating the filtered (F) solution.

Figure 2 compares the total (modeled + resolved) turbulent kinetic energy in all three cases compared to DNS of and RANS $k-\omega$ SST. It is well known that the RANS $k-\omega$ SST poorly predicts k. The hybrid RANS, with and without commutation error, improve this prediction. The cases (B and C) with commutation error, do even better than the hybrid case (A) without commutation error.

Figure 3 compares the repartition between modeled and resolved turbulent kinetic energy, in each of the three cases A; B and C. Interestingly, in case A, there is some residual fluctuating motion towards the wall, probably produced by the strong shear stress in this region of the flow. In contrast, cases B and C, using commutation error estimation, keep an almost zero resolved fluctuation level in this region where the target energy ratio r_k^t is 1 (RANS mode). Moreover, the log-layer mismatch, reflected by the discrepancy in peak locations, is reduced when using grey area mitigation strategies, like cases B and C.

Similarly, Figure 4 compares the *targeted* energy ratio r_k^t and the *observed* energy ratio r_k^o in all three cases. As mentioned above, cases B and C enforce a better RANS mode than case A, at the wall. However, outside the RANS area ($y^+ \geq 90$), all three strategies manage to force r_k^o towards r_k^t. This shows the efficiency of enforcing r_k, which is not natural in sHRL (see e.g. [10])

7 Conclusion and future work

A method has been presented for the mitigation of grey areas in seamless hybrid RANS/LES methods. Its theoretical background is the additive filter of Germano. The novelty of the present approach is that it assumes a blending between ensemble average (RANS) and identity (DNS), which greatly simplifies the overall procedure. While an ensemble-averaged quantity is reasonably easy to obtain, estimating the DNS field is a much harder challenge.

The results presented are a first step, not to say a proof of concept. They are contrasted: the proposed commutation error seems to have a bad effect on first moments like the streamwise velocity, but in the same time it slightly improves the ratio between resolved and modeled turbulent kinetic energy. The most obvious point for improvement is the estimation of the DNS field.

Planned future work includes to better estimate the F (DNS) fields, and to test the approach over

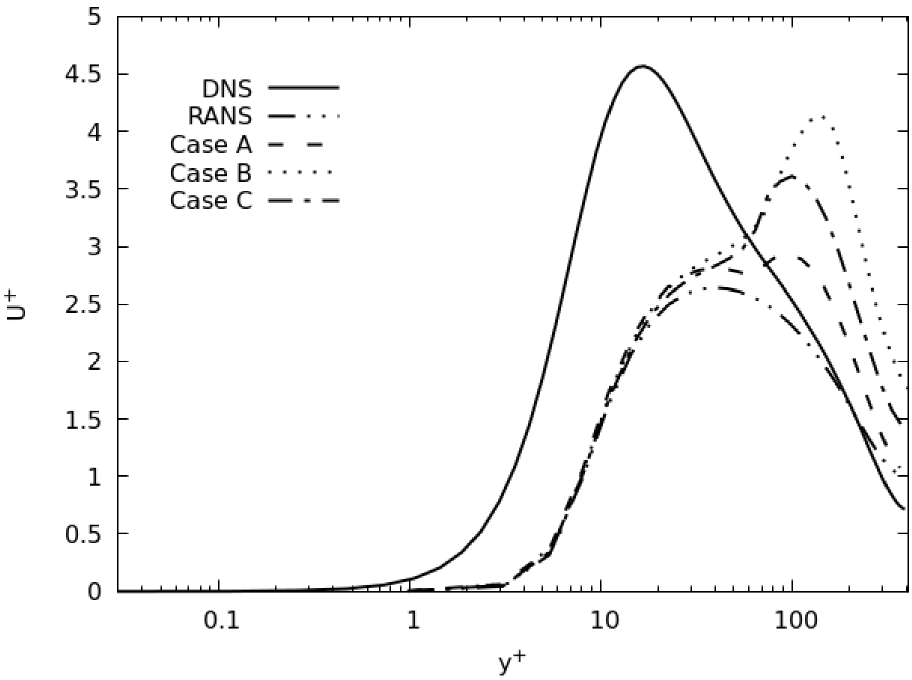

Figure 2: Total turbulent kinetic energy

other flows, such as the boundary layer developing over a flat plate (c.f. [8]), which implies a RANS-to-LES transition in both wall-normal and streamwise directions.

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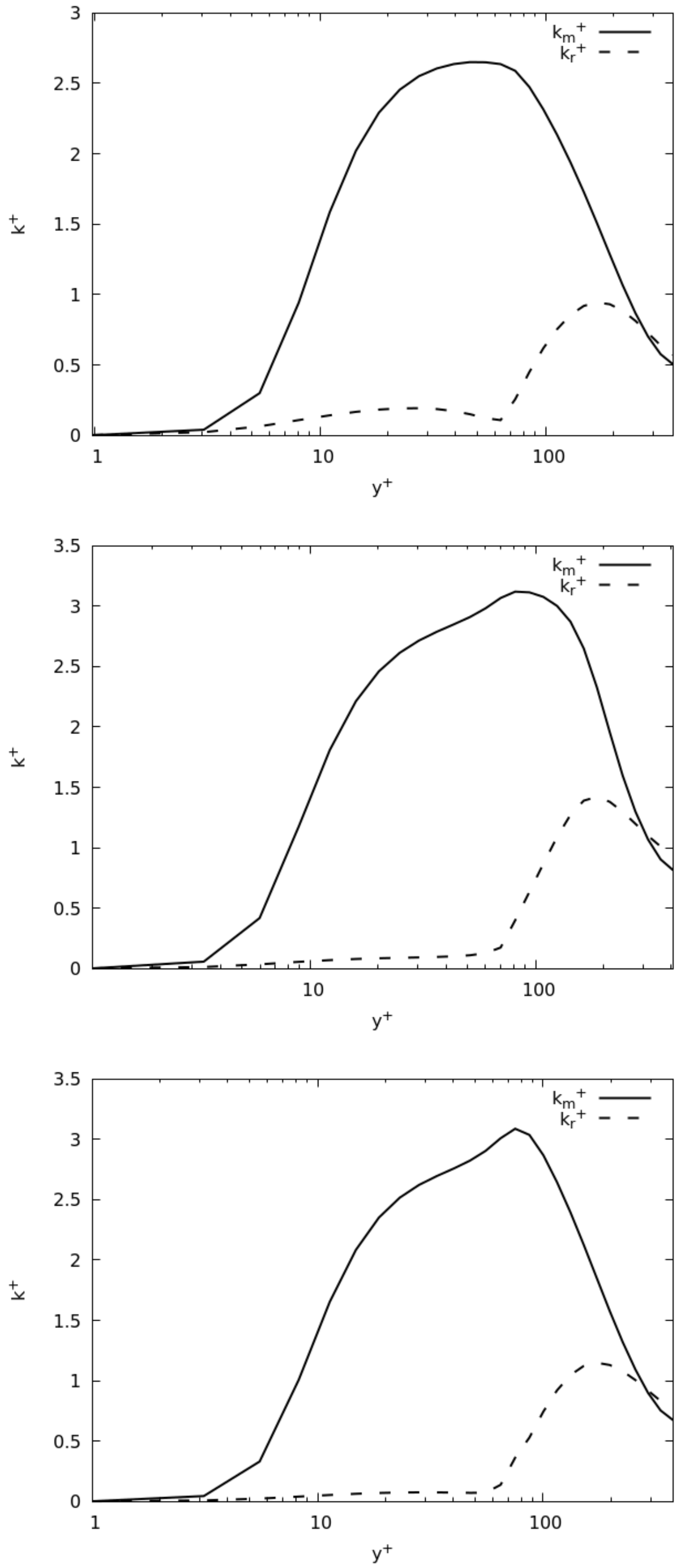
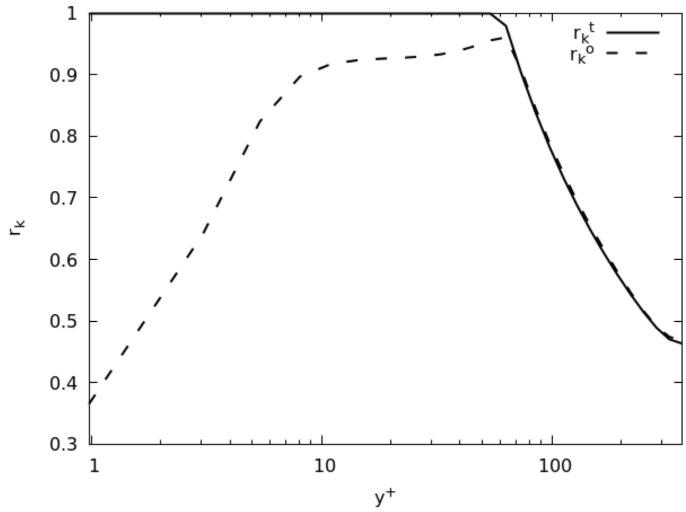
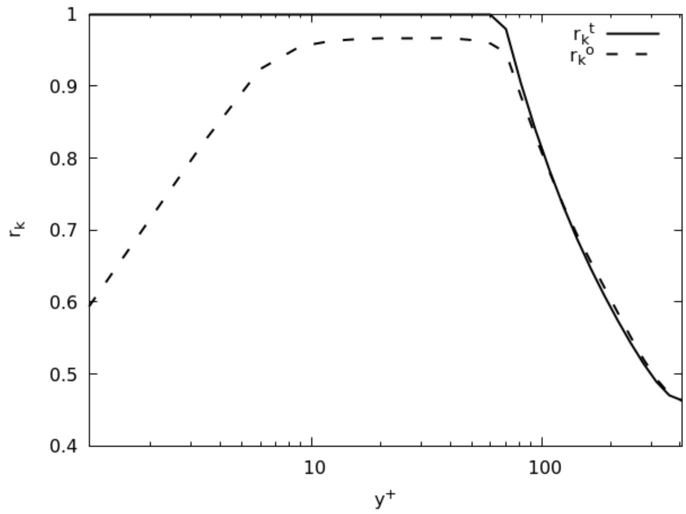


Figure 3: Repartition of turbulent kinetic energy between resolved (k_r^+) and modeled (k_m^+) counterparts. From top to bottom: case A, case B, case C.





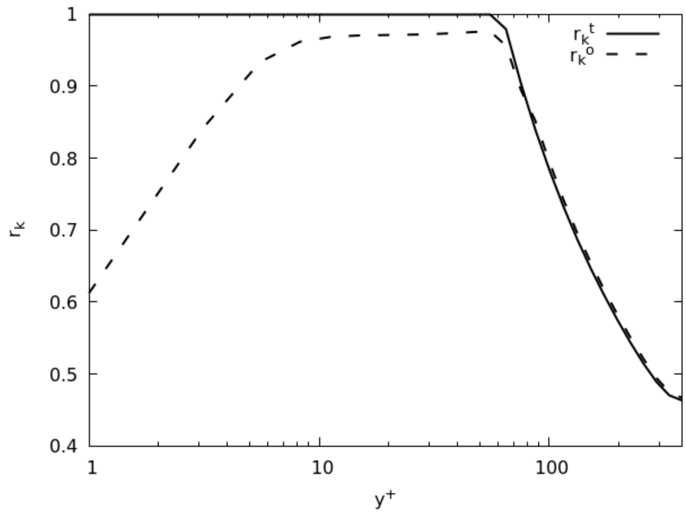


Figure 4: Comparison of *targeted* and *observed* energy ratios. From top to bottom: case A, case B, case C.