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[11-C-01] Representation of Small-Scale Spatiotemporal Nonlinear Dynamics by Low-dimensional Manifold

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Representation of Small-Scale Spatiotemporal Nonlinear Dynamics by Low-dimensional Manifold

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1 Introduction

We present a data-driven methodology for representing spatiotemporal nonlinear chaotic dynamics using deep neural network, in which the master dynamics in a low-dimensional space are employed to represent the slaved dynamics in the corresponding complemented subspace. It is shown that the proposed method comprehensively elucidates the underlying interaction mechanism in a nonlinear system where the small-scale dynamics is in fact completely slaved by the large-scale dynamics, as modelled by the theory of invariant and inertial manifolds. This then servers as a basis for effectively representing chaotic or turbulent systems by data in low-dimensional space. Theoritically, the method is supported by the representation of the dissipative dynamics, which reveals that the unresolved dynamics is slaved by the low-dimensional master dynamics. We finally demonstrate that the proposed method shows strong generalization and fidelity for spatiotemporal nonlinear dynamic systems.

In the realm of science and engineering, spatiotemporal nonlinear dynamics refers to the study of complex systems where both spatial and temporal dynamics play crucial roles. These systems often exhibit intricate patterns and interactions across multiple dimensions, making their analysis and prediction challenging. Chaotic behaviours are commonly observed in spatiotemporal systems, characterized by extreme sensitivity to initial conditions and the absence of long-term predictability. Understanding and predicting these chaotic behaviours is essential for various applications, including weather forecasting, ecological modelling, and understanding the behaviour of turbulent flows. To tackle the complexity of these systems, model reduction techniques have been employed to simplify the underlying dynamics while preserving the essential features. By identifying dominant modes or variables and capturing their interactions, reduced models enable efficient computation and provide valuable insights into the low dimensional dynamics governing the system. This reduction not only aids in unravelling the fundamental principles and behaviours of complex systems but also facilitates the design of control strategies and optimization in engineering applications.

In the study of spatiotemporal nonlinear dynamics, two dominant mode reduction strategies have been widely used throughscale decomposition. These strategies include both deterministic and stochastic methodologies. The decomposition of scales simplifies the analysis and forecasting of spatiotemporal systems, allowing for a more profound comprehension of the interaction between distinct scales and harnessing the complex behaviour of spatiotemporal nonlinear dynamics in various scientific and engineering fields. However, a majority of existingliterature takes small-scale dynamics as a stochastic process, using classical or ML-assisted low-dimensional modeling methods to eliminate small-scale dynamics and derive equations for large-scale dynamics exclusively. Although successful in establishing equations for large-scale dynamics, a notable challenge emerges in the effective capture and representation of small-scale dynamics. This discrepancy raises a crucial question regarding the production of a representation of fine-scale dynamics based solely on the information derived from large-scale dynamics. The investigation of methods to accurately represent small-scale dynamics and recover them exclusively from large-scale dynamics has not been undertaken in the existing literature, posing an open problem and a significant challenge.

An approximate inertial manifold, as highlighted earlier, offers a simplified representation of a system's long-term behavior by identifying a reduced set of variables or modes governing its evolution. This approach is utilized to approximate the interactionbetween large-scale and small-scale dynamics. Notably, modal expansion coefficients are distributed randomly in mode order and involve a wide range of spatial scales. However, it is still a chanllenge to find a single time scale for the evolution of each mode, especially for the higher mode numbers. Given the complexity, neural network is employed as a tool to establish such a high-fidelity mapping. Computationally, the construction of approximate inertial manifolds involves the nonlinear Galerkin procedure. The establishment of a slaved dynamics representation is inspired by the principles of the nonlinear Galerkin method, which forms the theoretical foundation for neural network approximation in higher mode space. This conceptual fusion introduces a novel methodology for representing small-scale dynamics and provides fresh insights into the connections between large-scale and small-scale dynamics, advancing our understanding of complex spatiotemporal nonlinear systems. Looking ahead, this innovative approach holds significant potential to recover the real dynamics of complex systems from low precision and fidelity dynamics alone. It opens avenues for further exploration and application, promising a deeper comprehension of intricate system behaviors in spatiotemporal nonlinear dynamics.

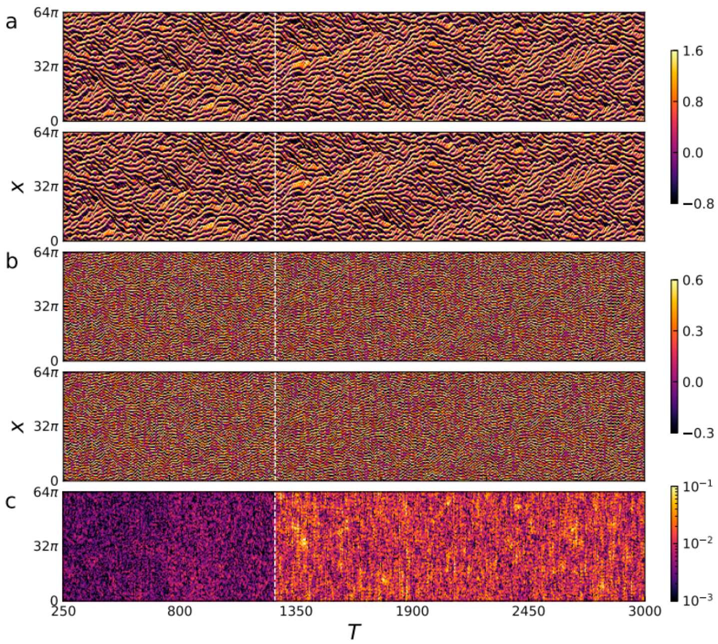

Figure 1: Illustration of dynamics of the Kuramoto-Sivashinsky equation in the full space and complementary manifold. a. Real full dynamics (Top) and represented full dynamics consist of real large-scale dynamics and represented slaved small-scale dynamics (Bottom). b. Real slaved small-scale dynamics (Top) and represented slaved small-scale dynamics (Bottom). c. Absolute error of small-scale slaved dynamics between truth and prediction. Left hand side and right hand side of vertical lines indicates corresponding quantities respectively on training dataset and test dataset.

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