Oral presentation | Turbulence simulation (DNS,LES,RANS)

Turbulence simulation(DNS,LES,RANS)-IV

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[11-B-03] Computational Modelling of Heat Convection using High Prandtl Number Fluids

*Haotian He[1], Ningshu Li[1], Hector Iacovides[1], Timothy J Craft[1] （1. Thermo-Fluids Group, Faculty of Science and Engineering, The University of Manchester, Manchester, UK）

Keywords: High Prandtl Number, RANS, Heat Convection, Numerical Method, Turbulence flow

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Computational Modelling of Heat Convection using High Prandtl Number Fluids

Haotian He*, Ningshu Li*, Hector Iacovides* and Timothy J Craft*
Corresponding author: haotian.he-2@student.manchester.ac.uk * Thermo-Fluids Group, The University of Manchester, Manchester M13 9PL, UK.

Abstract:
The paper analyses the performance of a number of standard and extended Reynolds-averaged Navier-Stokes (RANS) models with high Prandtl number fluids ($1 < Pr < 400$) through computations of forced, natural and mixed convection channel flow cases. The models tested include the high-Re $k-\varepsilon$, SST $k-\omega$, Launder-Reece-Rodi (LRR), the low-Re $k-\varepsilon$ of Launder-Sharma (LS), the same model including the lengthscale correction of Yap (LS_YAP) and the damping term modification of Sarno et al. (LS_MOD). Based on their performance, further modifications have been developed to the damping term in the LS form and the lengthscale correction (LSMOD_YAPMOD) to improve the predictions in high Pr cases. For forced convection, results showed that at high Prandtl numbers the proposed modified model (LSMOD_YAPMOD) captured the correct effect on the thermal field near the wall, showing very good quantitative agreement with DNS data. For natural convection, results revealed that the LSMOD_YAPMOD model accurately predicted both the dynamic and thermal fields in these natural convection flows across a range of moderate to high Prandtl numbers.

Keywords: High Prandtl Number, RANS, Turbulence Modeling, Forced, Natural and Mixed Convection

1 Introduction

Molten salt is widely recognized as a highly promising coolant, particularly in the realm of nuclear energy, where it finds extensive application in civil nuclear power reactors. The MSR was one of six reactor concepts proposed at the Fourth Generation International Forum (GIF IV) in 2001 [1]. MSRs are liquid-fueled reactors that use fuel dissolved in molten salt coolant, offering advantages such as high operating temperatures and low operating pressures. In contrast to traditional coolants, molten salt boasts superior thermodynamic and thermophysical properties, including low vapour pressure, exceptional thermal stability, and a wide operational temperature range.

A key characteristic of molten salts as coolants is their high kinematic viscosity (viscous diffusivity), resulting in a high Prandtl number. It possesses a high volumetric heat capacity and exhibits low molecular viscosity, addressing several design challenges and safety concerns associated with conventional reactor coolants. A detailed investigation of the heat transfer mechanisms in high Prandtl fluids is needed to resolve problems arising in various heat transfer systems and improve their behaviours. As a part of this, it is necessary to investigate the stability and accuracy of a variety of turbulence model predictions for high Prandtl fluids.

RANS modelling is commonly employed in the industry due to its relatively low computational cost. Despite its widespread use, the RANS method has limitations in providing entirely accurate and stable predictions because it relies on turbulence models and empirical parameter values. Therefore, it is beneficial to compare RANS with more accurate methods such as Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES).

The primary objective of this research is to investigate the flow and heat transfer characteristics of high Prandtl number fluids using RANS methods across various test cases. Further investigations will focus on improving the RANS modelling of turbulent heat convection in high Prandtl number fluids, with the aim of developing reliable and cost-effective models tailored for this purpose. The modified model proposed includes modifications to the damping term in the LS form and the lengthscale correction. All the models are implemented in the open-source CFD software OpenFOAMv-2206.

2 Turbulence Modelling

The tested models are widely available in many CFD software packages. These turbulence models are classified into two categories Eddy Viscosity Models (EVMs) or Reynolds Stress Transport Models (RSMs) to approximate the turbulent Reynolds stress tensor. The EVMs reproduce the mixing effects of large eddies through a turbulent viscosity, ν_t, while the RSMs solve individual transport equations for each component of the Reynolds stress tensor.

In the realm of two-equation EVMs, the $k-\varepsilon$ model is widely utilized in industry. As the name implies, this model determines the distribution of ν_t based on the distribution of turbulent kinetic energy, k, and its dissipation rate, ε. The high Reynolds number standard $k-\varepsilon$ model [2], suitable only for fully turbulent flows, is coupled with a wall function to circumvent the requirement of integrating equations all the way to the wall surface. The objective of the wall function strategy is to position the near-wall node far enough from the wall to be in the fully turbulent region ($y+ > 30$).

A low Reynolds number version of the $k-\varepsilon$ model is required in the present work. Such a version should include additional viscous damping terms and must resolve the steep gradients across the near-wall sub-layer. The low-Re models extend the applicability of high-Re models into the viscous sub-layer. These models address the over-prediction of turbulent kinetic energy levels near walls by fully integrating the equations up to the wall, which otherwise would result in excessive length scale values. Consequently, the mesh near walls must be very fine, and the near-wall distance y^+ should be kept below unity for the near-wall nodes. Notably, for fluids with $Pr \gg 1$, such as molten salts, the thermal boundary layer is very thin. Therefore, a substantial quantity of nodes was employed in the wall-normal direction of the computational mesh. For each high Prandtl number value considered in the present work, a mesh was generated to ensure that the location of the initial node complies with the condition of $y^+ \leq 0.01$. Four low-Re number EVMs are used in the present work, namely the Shear Stress Transpor (SST) $k-\omega$ [3], low-Re $k-\varepsilon$ of Launder-Sharma (LS) [4], the same model including the lengthscale correction of Yap [5] (LS_YAP) and the damping term modification of Sarno [6] (LS_MOD).

The LS model solves all transport equations numerically across the near-wall sub-layer. To improve its performance in computing flows with high Prandtl number fluids, Sarno et al. [6] proposed modifying the damping function, f_μ, in the turbulent viscosity $\nu_t = c_\mu f_\mu k^2/\varepsilon$, with an additional factor dependent on the turbulent Reynolds number, $Re_t = k^2/\nu\varepsilon$. The modified damping function term in the LSMOD model can be written as;

$$f_\mu = \exp\left[\frac{-3.4}{\left(1+\frac{Re_t}{50}\right)^2}\right]\cdot A$$
$$A = \begin{cases} 2.5/\left(Re_t^{0.3}\right), & \text{if } Re_t < 15 \\ 2.5/(Re_t^{0.3}), & \text{if } Re_t \geq 15 \end{cases}$$

As demonstrated by them, this version of the LS model yields more precise heat transfer predictions, particularly at higher Prandtl numbers. It increases the very near-wall eddy viscosity slightly (roughly for $y^+ < 10$) by decreasing the strength of the damping term. The effect on the dynamic field, and the thermal field for cases with Prandtl number around 1, was reported to be negligible, while it improved the heat transfer predictions for Prandtl numbers up to around 8.

To improve the performance of the LS model in recirculating flow, a source term, S_ε, was added to the transport equation for ε. This so-called "Yap correction" [5] reduces the turbulent length scale ($k^{3/2}/\varepsilon$) towards its local equilibrium value ($c_l y_w$). The Yap correction in the LS_YAP model can be expressed as;

$$S_\varepsilon = 0.83\frac{\varepsilon^2}{k}max[(\frac{\ell_t}{\ell}-1)(\frac{\ell_t}{\ell})^2, 0] \tag{1}$$

where ℓ_t is the turbulent length scale $k^{3/2}/\varepsilon$, the equilibrium length scale $\ell = 2.55y_w$, and y_w is the distance to the wall.

An alternative two-equation low-Re turbulence model considered in the present work is the $k-\omega$ Shear Stress Transport (SST) model. It solves a transport equation k and replaces the dissipation rate (ε) equation with one for the specific dissipation rate, $\omega = \varepsilon/c_\mu k$. It designed to combine the strengths of both the $k-\varepsilon$ and $k-\omega$ models. This hybrid approach utilizes the $k-\omega$ model in the near-wall region and the $k-\varepsilon$ model in the fully turbulent region, aiming to overcome the limitations of each model.

For high-Reynolds-number version RSMs, the Isotropization of the Production model by Launder, Reece, and Rodi (LRR-IP) [7] was tested in the present work. This model aims to resolve the RANS closure problem by solving transport equations for each component of the Reynolds stress tensor. It is designed to capture the complex interactions within a fluid flow by expressing the six exact stress productions resulting from mean straining, although other terms in the transport equations do require modelling.

The performance of these models is analyzed against available DNS and experimental data. Based on their performance, the present work developed further modifications to the damping term in the LS form and the lengthscale correction (LSMOD_YAPMOD) to improve the predictions in high Pr cases. The reasons and results will be discussed in the following sections.

All computations have been performed by employing a version of the open-source software OpenFOAM which has been extended to include the damping function and lengthscale modifications. To ensure a steady-state flow, all computations ran until the solution residuals are at least $\mathcal{O}(10^{-12})$, with the complete elimination of any variation in the streamwise direction. For all cases, grid resolutions were arrived at after the grid independence tests with at least three refined grids to ensure the model accurately captures heat transfer in the relatively thin boundary layer.

3 Results and Discussion

3.1 Forced Convection Flows

For the first forced convection simulation, the case follows Kozuka et al. [8] DNS study, involving a two-dimensional fully developed channel flow of incompressible fluids at $Re_\tau = 395$. Fluids of moderate Prandtl number values, $Pr = 0.7, 5, 10$, are considered in the case. The channel is long enough in the streamwise direction to ensure that the flow field becomes fully developed in the downstream region. The constant bulk velocity and zero gradient pressure conditions are specified at the inlet of the channel, whereas zero gradient velocity and zero fixed pressure conditions are specified at the outlet. A uniform heat-flux heating boundary condition is imposed along the top and bottom walls with the non-slip boundary condition for momentum. The primary objective here is a baseline prediction and code validation exercise to reproduce the effect of different Re on heat transfer.

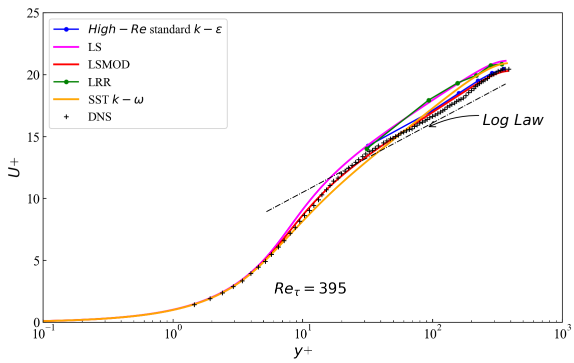


Figure 1: Non-dimensional velocity (U^+) profiles for fully developed channel flow at $Re_\tau = 395$; Comparison between high-Re standard $k-\varepsilon$, SST $k-\omega$, LS, LSMOD, LRR models, DNS data of Kozuka et al. [8].

For the prediction of mean flow features, mean velocity profiles (U^+) are shown in Fig 1. In general, the results obtained with high-Re $k-\varepsilon$, SST $k-\omega$ and the LSMOD models are in good agreement with the DNS data across the range of Re_τ. Both LS and LRR models slightly overpredict the mean velocity, which corresponds to the underprediction of turbulence level within the log-law region ($y^+ \geq 30$). All three EVMs, LS, LSMOD and SST models, are in excellent agreement in the viscous sublayer $y^+ \leq 5$ when compared with the DNS. The LS model, compared to the LSMOD model, departs from the DNS data outside the viscous sublayer $y^+ \geq 5$. It is clear that the modified damping term in the turbulent viscosity of the LSMOD model does not significantly affect the dynamic field and offers little to no improvement in the accuracy of dynamic field predictions for low-Re $k - \varepsilon$ models.

For the low-Re $k - \varepsilon$ models, the increase in turbulence levels as the strength of the overall damping term in the turbulent viscosity decreases (as implied by the velocity profiles) is confirmed through examination of the non-dimensional eddy viscosity ν_t/ν, shown in Fig 2. In the near-wall region, the eddy viscosity magnitude significantly differs between both models. The higher ν_t/ν, produced by LSMOD, indicates that this modified damping term does indeed increase eddy viscosity near the wall.

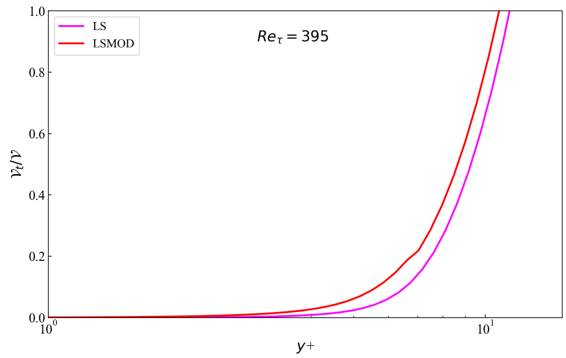


Figure 2: Eddy viscosity ratio profiles for fully developed channel flow with $Re_\tau = 395$. Comparison between LS and LSMOD models.

For the prediction of the thermal field, Fig 3 presents the Nusselt number comparisons (Nu) of all considered models across the range of Pr. It can be seen that for $Pr = 0.7$ and $Re_\tau = 395$, all models exhibit qualitatively similar performance but, for the LSMOD model, the introduction of the modified damping term results in a slight over-prediction of the Nu. Since the widely used turbulence models have been adjusted for fluids with a Pr close to unity in predicting the thermal field, this would be expected to be well represented by the unmodified turbulence models.

As Pr increases to 5 and 10, contrary to the above, the LSMOD model provides superior predictions of Nu. The LS model significantly under-predicts the Nu, making agreement with the DNS relatively poor. The comparisons also present that the difference in the Nu prediction between LS and DNS gradually increases as Pr increases (within 11% at $Pr = 5$, and within 18% at $Pr = 10$). This is consistent with previous observation obtained from ν_t/ν. The reduced strength of the damping term in the turbulent viscosity near the wall leads to an increase in turbulent viscosity. Consequently, this results in elevated turbulent diffusivity, ultimately yielding a higher Nu. Additionally, similar Nu predictions are demonstrated for wall function models between the high-Re standard $k - \varepsilon$ and the LRR models for all considered Pr cases.

For the second simulation, a two-dimensional fully developed channel flow with a constant wall temperature difference boundary condition is applied in the wall-normal direction. The chosen case study follows DNS studies by Schwertfirm et al. [9] and Hasegawa et al. [10] at $Re_\tau = 150$ and 180. High Pr number fluids, with a wide range of $Pr = 1,\ 10,\ 49,\ 100,\ 200,\ 400$, are considered. The influence of Pr on the thermal field is only investigated by employing the low-Re LS (1974) $k - \varepsilon$ both with and without the modified damping function term. Since Sarno et al. conducted simulations only at moderate Prandtl numbers up to 8, the aim here is to confirm that the accurate prediction of heat transfer discussed there can be further reproduced correctly for high Pr fluids. The tested models are the low-Re LS $k - \varepsilon$ model both with and without the modified damping function term. The boundary conditions remain consistent with those in the first channel case, with the only modification being the application of different constant temperatures to both walls.

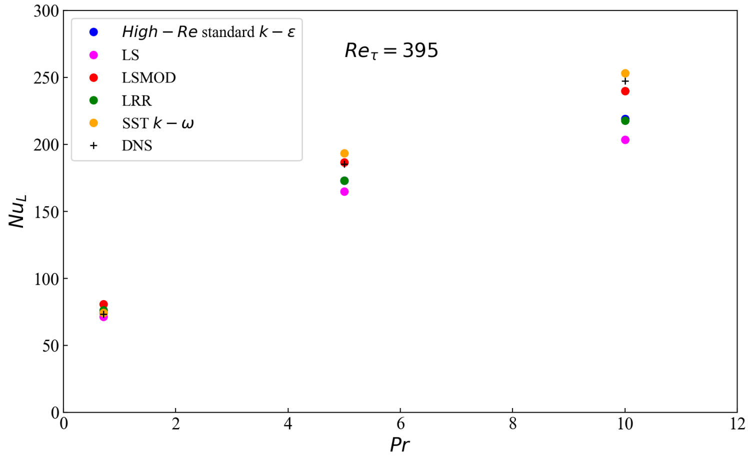
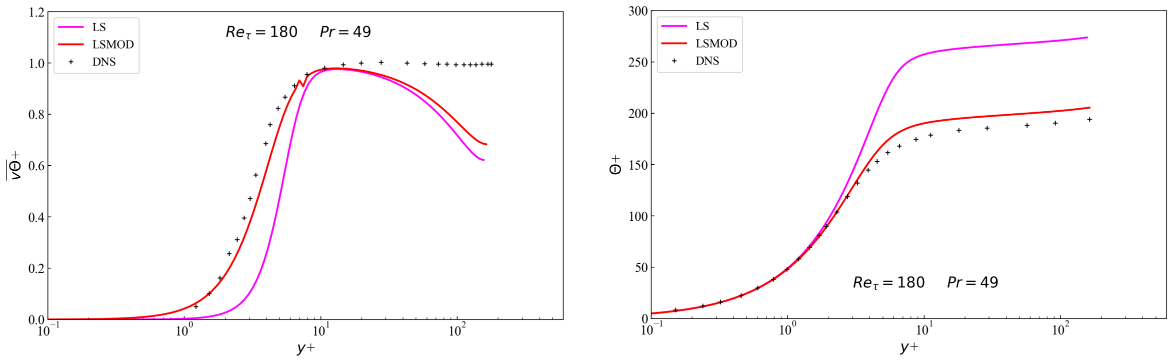


Figure 3: Nusselt number comparisons for channel flow over a range of Prandtl numbers; $Pr = 0.7,\ 5,\ 10$. Comparison between high-Re standard $k-\varepsilon$, SST $k-\omega$, LS, LSMOD, LRR models and DNS of Kozuka et al. [8]; $Re_\tau = 395$.



(a) (b)

Figure 4: Non-dimensional (a) turbulent heat flux; (b) mean temperature profiles for channel flow at $Pr = 49$ and $Re_\tau = 180$; Comparison between the LS, LSMOD models and DNS of Schwertfirm et al. [9].

For thermal field predictions, Fig 4 presents profiles of the wall-normal turbulent heat flux ($\overline{v\Theta}^+$) and mean temperature (Θ^+) of the LS and LSMOD models at $Pr = 49$ and $Re_\tau = 180$ as a reference case. Similar predictions were observed for cases with other Pr and Re_τ values and will not be shown here. In the near-wall region ($y^+ < 10$) for $\overline{v\Theta}^+$, it is evident that the LSMOD, incorporating the modified damping function term, exhibits good agreement with the DNS data. As mentioned, the inclusion of the modified damping term leads to an enhancement in eddy diffusivity by directly elevating the eddy viscosity in the near-wall region. Regarding the mean temperature profiles, significant difference between the predictions of the LS model and the LSMOD model are evident in both the near-wall and log-law regions. The LSMOD model demonstrates remarkable accuracy in predicting temperature profile behaviour.

The above observations are also confirmed in comparisons of the Nusselt number, as shown in Fig. 5. In general, the results obtained with the LSMOD model are in good agreement with the DNS data across the range of Pr. It also can be seen that the LSMOD model, with the modified damping function term, responds well to the increase in Pr. However, the comparisons present that the difference in the Nu computation between LS and DNS gradually increases as Pr increases (within 15% at $Pr = 10$, within 28% at $Pr = 49$, and within 37% at $Pr = 400$). Hence, the supplementary damping modifications incorporated in the LSMOD model exert a noteworthy impact on the Nu, particularly resulting in a substantial augmentation at higher Pr, such as $Pr = 400$. This suggests that at the high Prandtl number values the modification for the damping term in the eddy viscosity formulation substantially improves

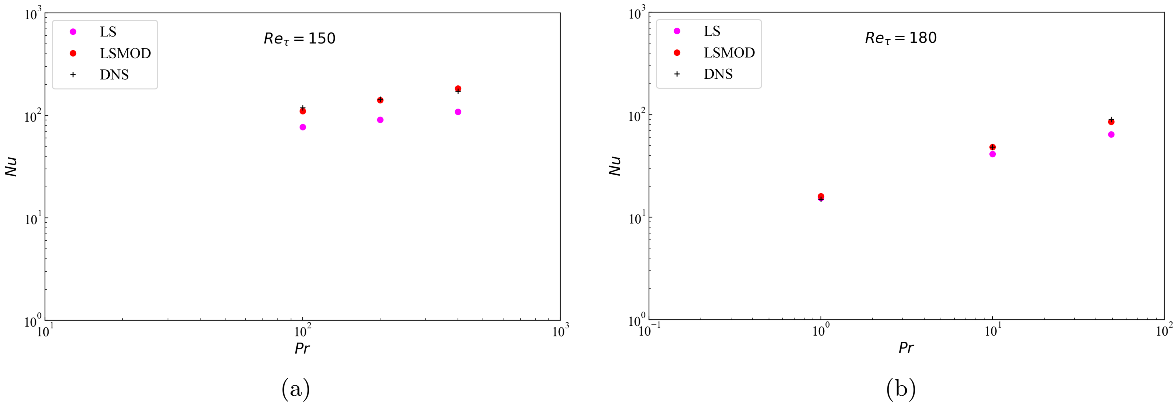


Figure 5: Nusselt number comparisons for channel flow over a range of Prandtl numbers at $Re_\tau = 150, 180$; Comparison between the LS, LSMOD models and DNS of Schwertfirm et al. [9] and Hasegawa et al. [10].

the thermal predictions. Comparisons between the predicted and the DNS of the thermal field across the wide range of high Prandtl numbers ($Pr = 1 \sim 400$) appear to be consistent with those at the moderate Pr in the first simulation case.

3.2 Natural Convection Flows

To explore the mentioned effects and assess the accuracy of numerical models for natural convection with high-Prandtl-number fluids, computations are carried out between two infinite, differentially heated, vertical flat plates. The case considered here is a two-dimensional vertical buoyancy-driven channel flow between two impermeable and isothermal walls. Periodic boundary conditions are imposed on velocity, pressure and temperature in the streamwise direction. No-slip conditions along with constant temperatures, are imposed on both wall surfaces, and the velocity field is initialized with zero values. The Rayleigh number $Ra = 10^8$ considered here is well within the turbulent regime for the natural convection case. Regarding the Prandtl number, the parameter range considered is $Pr = 1,\ 5,\ 10,\ 100$. The tested models here are the SST $k - \omega$, the low-Re LS $k - \varepsilon$ both with (LSMOD) and without (LS) the modified damping function term, and the LS model including the length scale correction of Yap (LS_YAP), as well as the LSMOD model including the length scale correction of Yap (LSMOD_YAP). The chosen case study follows the DNS study by Howland et al. [11].

Mean vertical velocity profiles predicted by all tested models are displayed in Fig 6. The profiles are scaled by the buoyancy ($U_{buo} = \sqrt{g\beta\Delta\Theta L}$) velocity. At $Pr = 1,\ 5,\ 10$, the SST $k - \omega$ model consistently over-predicts mean velocity, resulting in higher peak values compared to DNS data, indicative of an under-prediction of turbulence levels. Conversely, the mean velocity profiles of the LS and LSMOD models indicate under-predictions in the mass flow rate through the channel. This can be attributed to the over-prediction of turbulence levels. It is noteworthy that the incorporation of the modified damping function term in the LS model to reduce near-wall damping does not substantially impact the dynamic field. This observation aligns with the findings observed earlier in the previous section. The predictions obtained with the LS_YAP and LSMOD_YAP models exhibit relatively good agreement with the DNS data at $Pr = 1, 5, 10$ compared to the other models. The introduction of the Yap correction makes an improvement in the mean velocity at each Pr. Since the Yap source term acts to increase ε and subsequently reduce the length scale, primarily affecting the viscous sub-layer, it reduces turbulent kinetic energy, especially in proximity to the wall.

As Pr increases to 100, all models demonstrate qualitatively similar performance, showing similar peak values of vertical velocity. However, quantitative agreement with the DNS data remains relatively poor. Away from the side wall, all variations of the low-Re LS model struggle to accurately represent the change in slope after the velocity peak, resulting in a parabolic shape instead of a sharp straight line. As the Yap correction is sensitive to the proximity to the wall, its effectiveness is influenced by the distance to the wall. When Pr exceeds a critical threshold, leading to exceptionally thin thermal boundary layers which may become thinner than the viscous sub-layer, the Yap correction is rendered ineffective due to the diminishing distance from the wall to the peak vertical velocity. As a result, at $Pr = 100$, all models exhibit comparable performance.

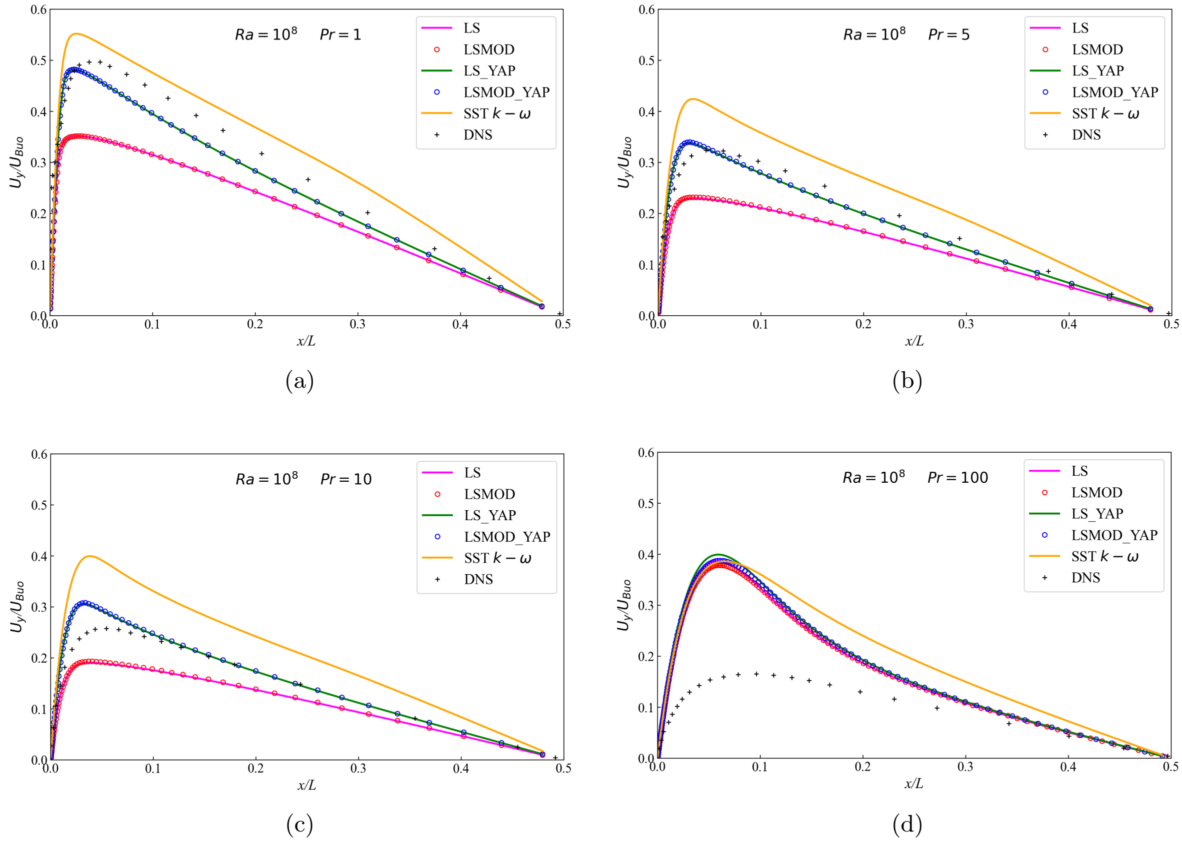


Figure 6: Mean vertical velocity profiles scaled by the buoyancy velocity (U_y/U_{Buo}) for natural convection channel flow with $Ra = 10^8$ and $Pr = 1,\ 5,\ 10,\ 100$; Comparison between LS, LSMOD, LS_YAP, LSMOD_YAP, SST $k-\omega$ models and DNS data of Howland et al. [11].

The above behaviour is confirmed by inspection of the corresponding turbulent length scale profiles. As documented in Ince et al. [12], when boundary layers are out of equilibrium, the $k-\varepsilon$ model is notorious for yielding excessively large near-wall length scales. Excessive length scales result in excessively high eddy viscosity. The Yap correction serves to guide and reduce the length scale towards its local equilibrium value.

The profiles of the turbulent length scale ratio (ℓ_t/ℓ) in Fig 7 illustrate that the LS_YAP model, incorporating the Yap correction, results in a reduced turbulent length scale near the wall compared to the LS model at $Pr = 1,\ 5,\ 10$. As Pr increases, the influence of the Yap correction diminishes gradually. Specifically, at $Pr = 100$, the turbulent length scale becomes significantly smaller than the equilibrium length scale due to the presence of both extremely thin thermal and dynamic boundary layers, rendering the Yap correction inactive and thus not contributing.

Non-dimensional turbulent viscosity distribution, particularly close to the wall, is shown in Fig 8. The k-ω SST model demonstrates the lowest near-wall turbulent viscosity among all five models, correlating with the highest predicted mean velocity in Fig 6. The modified damping function term has a nearly negligible impact on the turbulent viscosity in the near-wall region for these natural convection flows. A comparison of turbulent viscosity profiles between the LS and LS_YAP models reveals a notable reduction in turbulent viscosity in the near-wall region with the introduction of the Yap term, except at $Pr = 100$. By implementing the Yap correction amendment, the $k-\varepsilon$ model restricts the near-wall turbulent length scale growth from deviating significantly from the local equilibrium value, enhancing the accuracy of near-wall velocity profile predictions. This improvement is attributed to the decrease in turbulent viscosity levels, resulting in reduced turbulent kinetic energy and substantially higher laminar

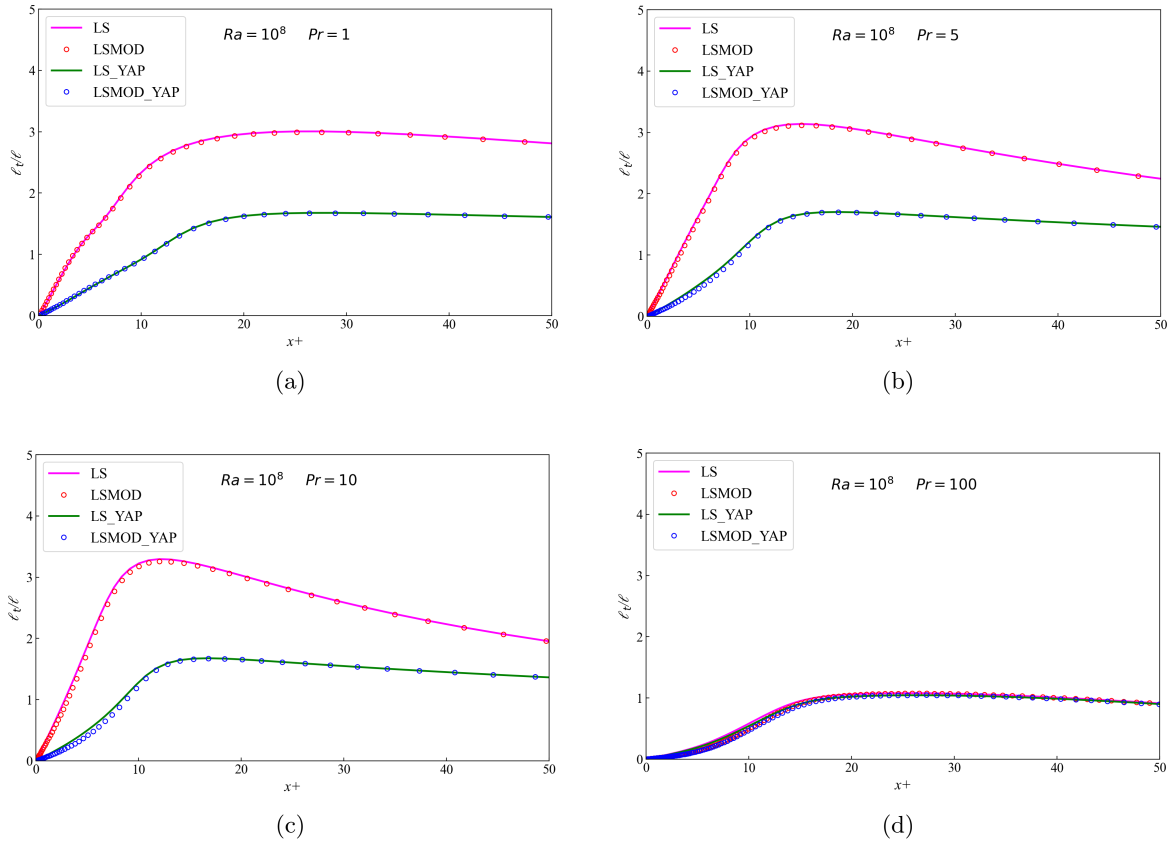


Figure 7: Turbulent length scale ratio profiles for natural convection channel flow at $Ra = 10^8$ and $Pr = 1,\ 5,\ 10,\ 100$; Comparison between LS, LSMOD, LS_YAP and LSMOD_YAP models.

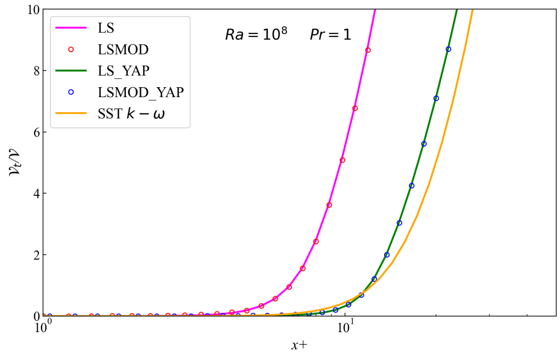
convective transport.

For the prediction of the thermal field, Fig 9 presents the mean temperature profiles of the five tested turbulence models at $Ra = 10^8$ and $Pr = 1,\ 5,\ 10,\ 100$. For the SST $k-\omega$, profiles indicate a notable overprediction of the mean temperature compared to the DNS data, consistent with the significantly lowest turbulent viscosity it yielded near the wall in Fig 8.

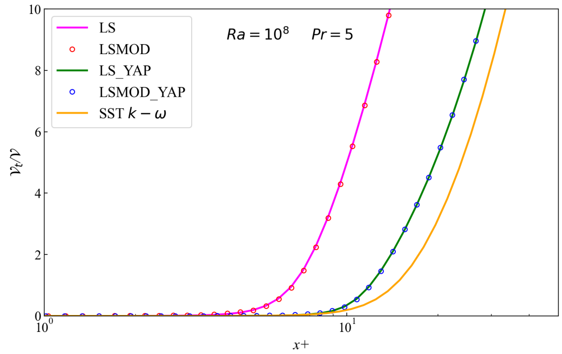
At the lowest $Pr = 1$, all test models exhibiting deviations from the DNS data are deemed acceptable. At moderate to high $Pr = 5,\ 10$, both the LS and LSMOD models exhibit good agreement with the DNS data. The Yap correction has a dominant effect on temperature prediction compared to the modified damping function term. Introducing the Yap correction in the LS model exacerbates the over-predictions in the temperature profile observed at moderate to high Pr, leading to poor quantitative agreement with the DNS data. This result is unsurprising, as the Yap correction directly decreases eddy viscosity in the near-wall region, thereby reducing eddy diffusivity. Consequently, turbulent heat flux decreases, resulting in an over-prediction of the mean temperature as compensation for the energy balance.

Generally, at the highest considered $Pr = 100$, the implementation of the modified damping function term, compared to the Yap correction, has minimal to negligible impact on temperature prediction in this natural convection flow. However, limited improvement is observed when the Yap correction is not activated at extremely high Pr.

Fig 10 shows the Nusselt number comparisons for each Pr. It is evident that the modified damping function term has a limited effect on Nu, as previously supported by Fig 9. At $Pr = 1$, the LS_YAP model shows a relatively good prediction of Nu, yet there remains a significant discrepancy with the DNS data. In contrast, at $Pr = 5,\ 10$, the introduction of the Yap correction over-corrects Nu, leading to an amplification of the error compared to the DNS data. This percentage error gradually escalates as Pr increases (within -16.3% at $Pr = 1$, 36.1% at $Pr = 5$ and 42.5% at $Pr = 10$ for the LS_YAP model).



(a)



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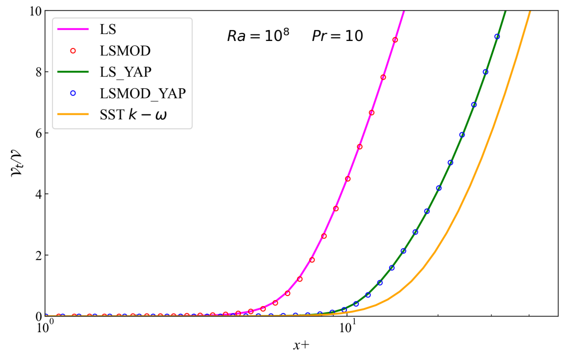

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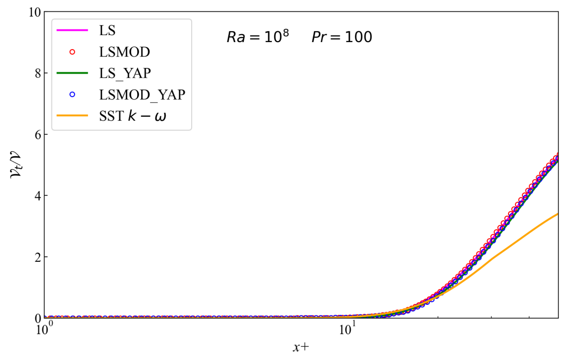


(d)

Figure 8: Eddy viscosity ratio profiles for natural convection channel flow at $Ra = 10^8$ and $Pr = 1,\ 5,\ 10,\ 100$; Comparison between LS, LSMOD, LS_YAP, LSMOD_YAP and SST $k-\omega$ models.

At $Pr = 100$, all models exhibit significant errors (approximately around -60%), diverging notably from the DNS data. This underscores that the Yap correction remains inactive due to the extremely thinner thermal boundary layers, a direct consequence of the higher Pr, where the thermal diffusivity is lower for a fixed Ra.

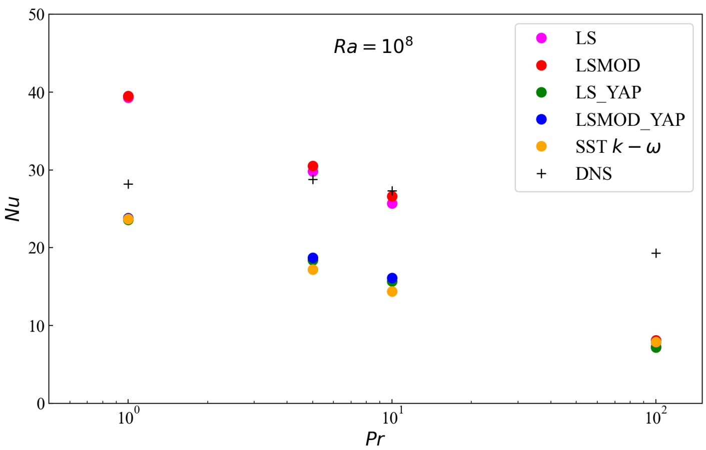


Figure 10: Nusselt number comparisons for natural convection channel flow over a range of Prandtl numbers $Pr = 1,\ 5,\ 10,\ 100$ at $Ra = 10^8$; Comparison between LS, LSMOD, LS_YAP, LSMOD_YAP, SST $k-\omega$ models and DNS data of Howland et al. [11].

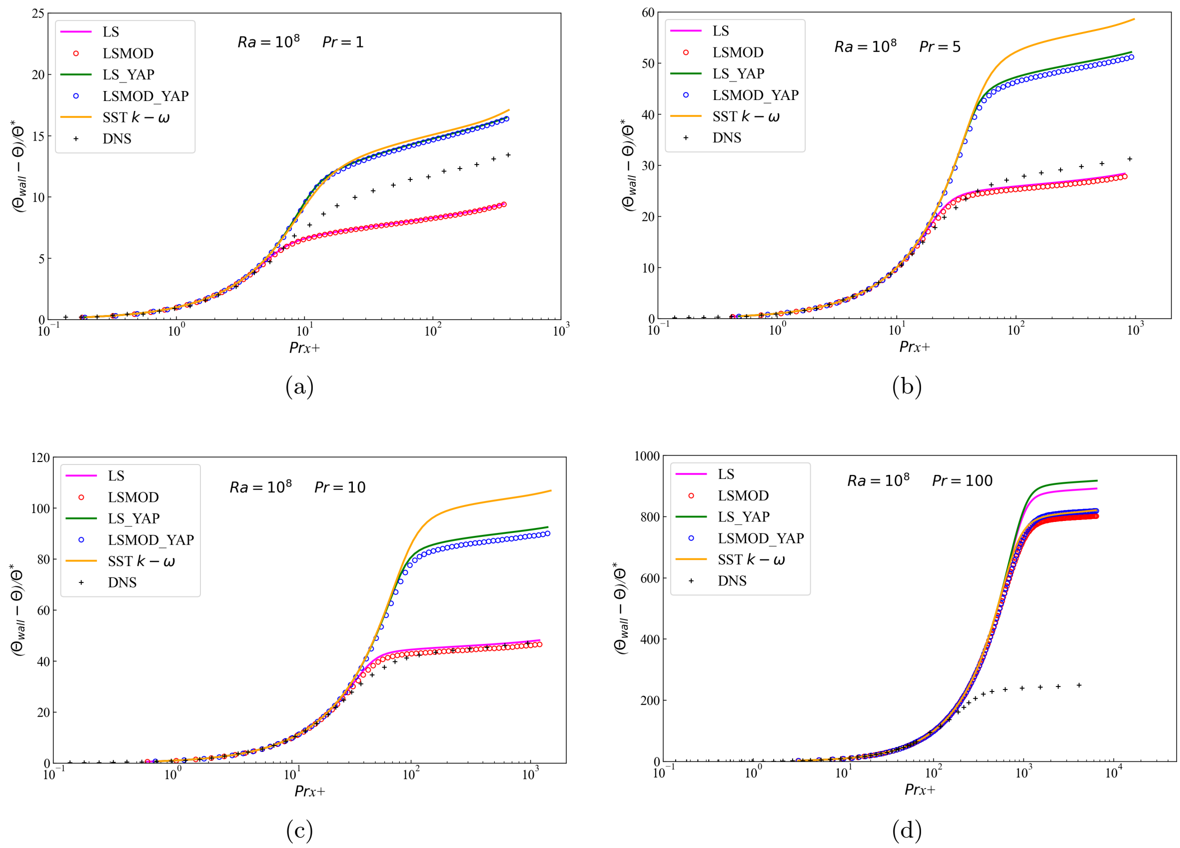


Figure 9: Mean temperature profiles scaled by the friction temperature ($(\Theta_{wall} - \Theta)/\Theta^*$), for natural convection channel flow at $Ra = 10^8$ and $Pr = 1, 5, 10, 100$; Comparison between LS, LSMOD, LS_YAP, LSMOD_YAP, SST $k - \omega$ models and DNS data of Howland et al. [11].

4 Further Model Modifications

From the above results, at moderate-high Pr, it is apparent that including the Yap term in the LS model amplifies the error and reduces the accuracy of the thermal field predictions in these natural convection flows. However, a significant improvement was observed with the Yap correction in the dynamic field prediction by increasing ε and consequently reducing the length scale.

To achieve a balanced performance of the low-Re $k - \varepsilon$ model for relatively accurate predictions of both dynamic and thermal fields in natural convection at moderate to high Prandtl number values. Thus, an alternative approach utilizing a function incorporating the damping term is proposed to adjust the turbulent viscosity. This adjustment aims to enhance the accuracy of predictions concerning dynamic and thermal fields for very high Prandtl number fluids. This approach is characterized by a reduced strength of the Yap correction and a modified turbulent viscosity as a function of the Prandtl number, and is outlined as follows:

$$ModYap = S'_\varepsilon = 0.83min[(\frac{Re_t}{500})^6, 1]\frac{\varepsilon^2}{k}max[(\frac{\ell_t}{\ell} - 1)(\frac{\ell_t}{\ell})^2, 0] \tag{2}$$

$$\nu'_t = C_\mu \frac{f_\mu k^2}{(1 - \exp(-2200(1 + Pr^2)) + \exp(-2200(1 + Pr^2))f_\mu)\varepsilon} \tag{3}$$

Additionally, to maintain numerical stability, a modification was introduced to the form of the coefficient A in the modified damping term.

$$Re_t < 15 : A = min(\frac{2.5}{Re_t^{0.3}}, 10) \tag{4}$$

In what follows, the Launder-Sharma $k - \varepsilon$ model with the inclusion of the modified Yap term in association with this modified turbulent viscosity damping term is referred to by the abbreviation LSMOD_YAPMOD.

The performance of the LSMOD_YAPMOD model is validated through examination of the corresponding velocity profiles, as depicted in Fig 11. For $Pr = 1$, it demonstrates that the influence of the modified Yap correction on the mass flow rate is relatively insignificant compared to the LSMOD_YAP model with the unmodified Yap correction. This outcome arises primarily from the sufficient distance between the wall and the peak vertical velocity. In the case of unity Pr, the buoyancy-induced temperature gradient occurs across a relatively extensive spatial domain, causing the peak vertical velocity to occur farther away from the wall. Although the modified Yap correction slightly increases turbulent viscosity in the near-wall region, adjusting turbulent viscosity solely in this region has a limited impact on the overall prediction of the dynamic field.

As Pr increases to 5 and 10 however, the profiles of the LSMOD_YAPMOD do show a qualitative under-prediction response. This can be attributed to the slight increase in near-wall turbulent viscosity induced by the modified Yap correction, aimed at enhancing the prediction of the thermal field. The gradual thinning of the temperature boundary layer leads to peak vertical velocity occurring closer to the wall. Thus, the modification of the Yap correction near the wall exerts a notable influence on velocity predictions for the LSMOD_YAPMOD model compared to the prediction at $Pr = 1$. The profile of the LSMOD_YAPMOD model at $Pr = 100$ shows significant improvement and demonstrates good agreement with the DNS data.

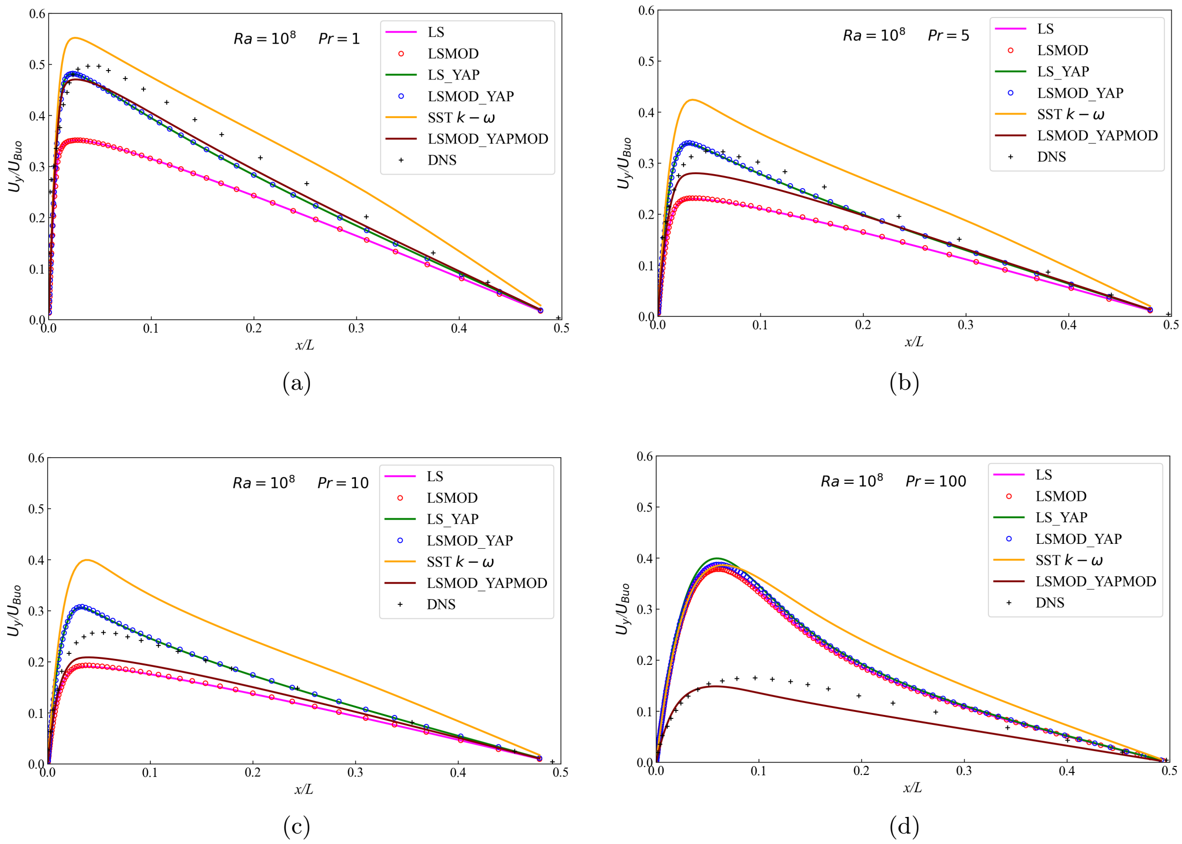
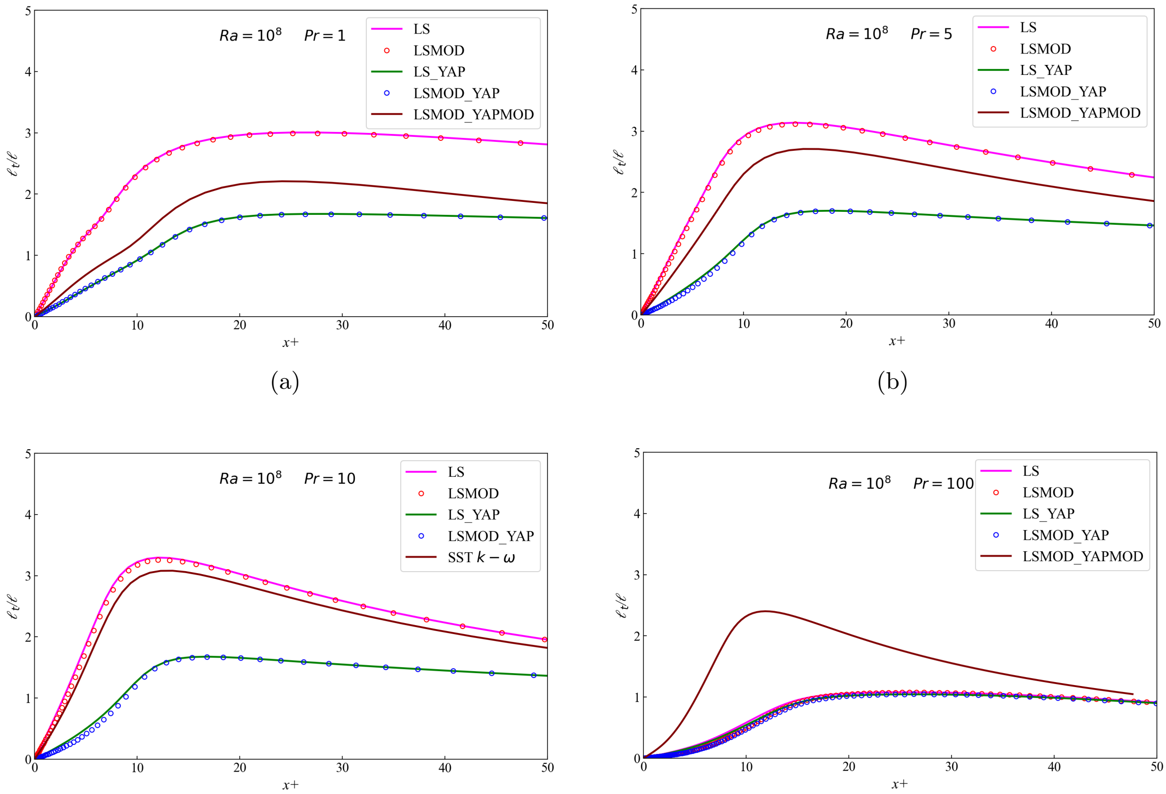


Figure 11: Mean vertical velocity profiles scaled by the buoyancy velocity (U_y/U_{Buo}) for natural convection channel flow with $Ra = 10^8$ and $Pr = 1,\ 5,\ 10,\ 100$; Comparison between LS, LSMOD, LS_YAP, LSMOD_YAP, SST $k-\omega$, LSMOD_YAPMOD models and DNS data of Howland et al. [11].

The above observations are supported by Fig 12 and Fig 13 which present profiles of ℓ_t/ℓ and ν_t/ν, respectively. Comparing the profiles of the LSMOD_YAPMOD model to those of the LSMOD_YAP model, it can be seen that the modified Yap correction causes an increase in both turbulent length scale and eddy viscosity near the wall. It is clear that as Pr increases, the discrepancies between the

LSMOD_YAPMOD and LSMOD models in both of these quantities gradually decrease, indicating a decreasing contribution from the modified Yap correction until it has almost no effect on the length scale and turbulent viscosity distributions by $Pr = 10$. At $Pr = 100$, both these profiles using the LSMOD_YAPMOD model present an increase compared to the corresponding profiles of the other models. These observations are consistent with the vertical velocity profiles shown in Fig 11.

As was seen earlier in Fig 11, the modifications to the Yap correction as part of the LSMOD_YAPMOD model result in a reduction of the vertical velocity. In some cases and regions of the flow, this does lead to a reduced quantitative agreement with the DNS data. However, this is a result of trading off accuracy in the dynamic field predictions with improving the thermal field predictions, as will be seen below.



(a) (b) (c) (d)

Figure 12: Turbulent length scale ratio profiles for natural convection channel flow at $Ra = 10^8$ and $Pr = 1,\ 5,\ 10,\ 100$; Comparison between LS, LSMOD, LS_YAP and LSMOD_YAP, LSMOD_YAPMOD models.

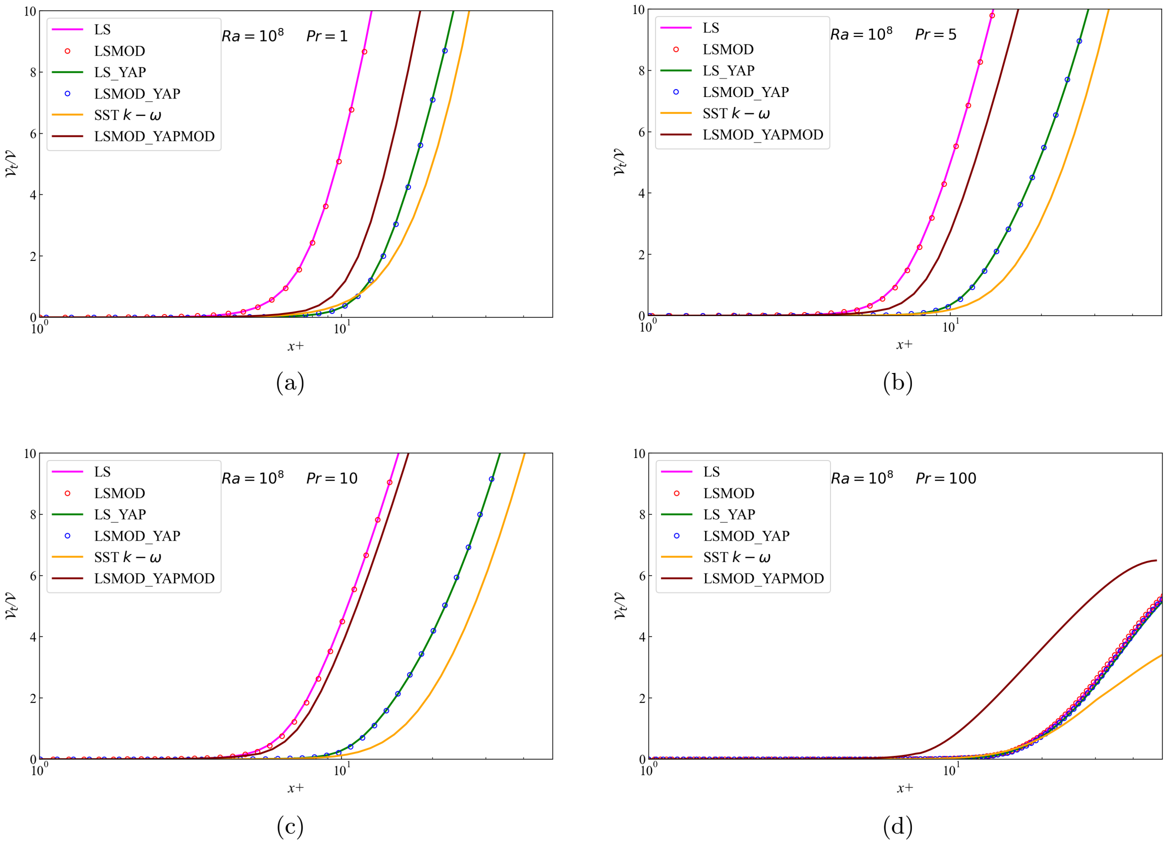


Figure 13: Eddy viscosity ratio profiles for natural convection channel flow at $Ra = 10^8$ and $Pr = 1,\ 5,\ 10,\ 100$; Comparison between LS, LSMOD, LS_YAP, LSMOD_YAP and SST $k-\omega$, LSMOD_YAPMOD models.

For thermal field predictions, the mean temperature profiles of the LSMOD_YAPMOD model presented in Fig 14 show improvements across the range of moderate to high Pr. The implementation of the modified Yap correction results in a slight increase of eddy viscosity near the wall compared to the unmodified version, thereby increasing eddy diffusivity and turbulent heat flux. This subsequently leads to a slight reduction in mean temperature as compensation for the energy balance, ultimately achieving a relatively good agreement with the DNS data. It is notable that as Pr increases, the diminishing contribution of the modified Yap correction aligns with the observations mentioned above. At $Pr = 100$, despite the limited impact of the Yap correction and the modified damping term in the presence of an extremely thin thermal boundary layer, the employment of the modified eddy viscosity notably improves the qualitative agreement of prediction by the LSMOD_YAPMOD model with the DNS data.

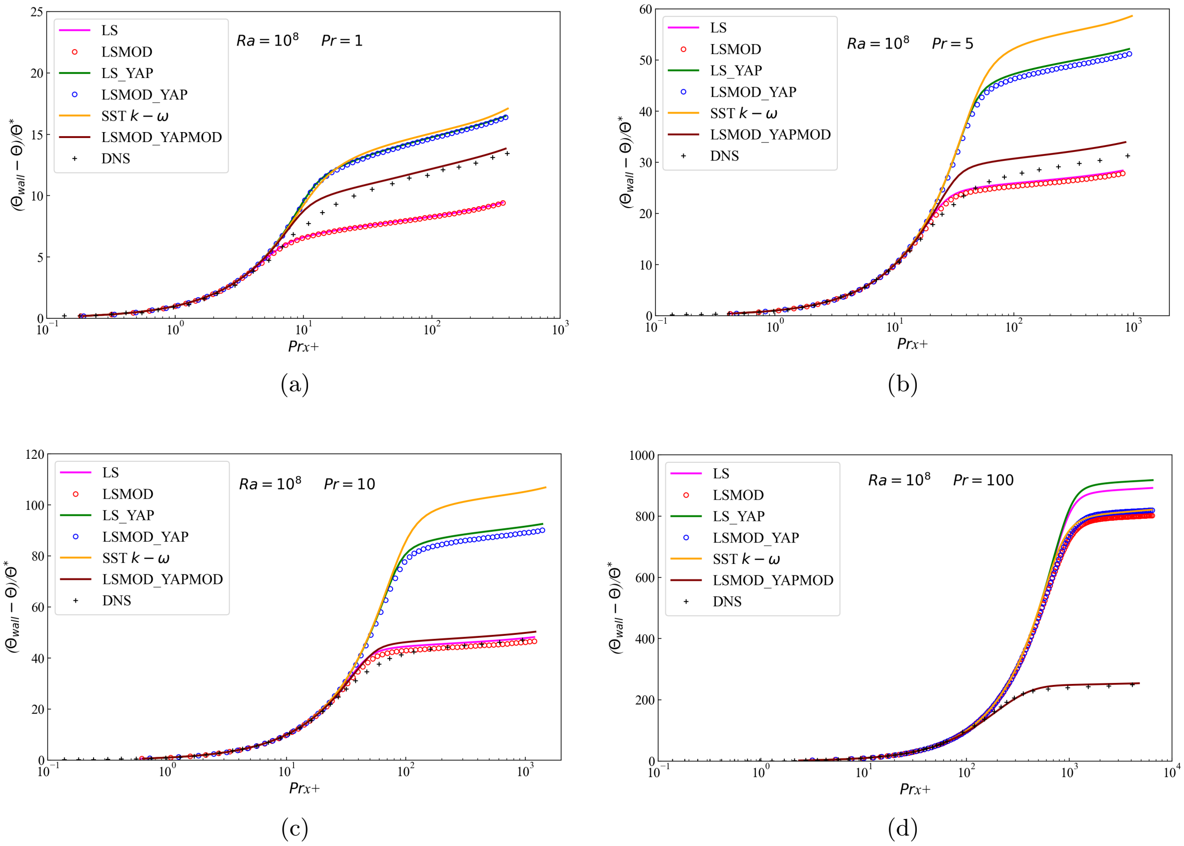


Figure 14: Mean temperature profiles scaled by the friction temperature ($(\Theta_{wall} - \Theta)/\Theta^*$), for natural convection channel flow at $Ra = 10^8$ and $Pr = 1, 5, 10, 100$; Comparison between LS, LSMOD, LS_YAP, LSMOD_YAP, SST $k - \omega$, LSMOD_YAPMOD models and DNS data of Howland et al. [11].

Fig 15 provides comparison of the Nusselt number against DNS data at each Pr. As expected, the LSMOD_YAPMOD model generally shows close agreement with the DNS data. At $Pr = 1$, the LSMOD_YAPMOD model with the weakened Yap correction presents an excellent prediction of the Nu, with the error reduced from -15.6% to 0.2% compared to employing the unmodified Yap correction. At $Pr = 5$ and $Pr = 10$, the performance of the LSMOD_YAPMOD model in predicting the Nu is also improved, with the error reduced from -35.1% to -8.7% and -41.0% to-7.7%, respectively. Although a slight discrepancy persists compared to the DNS data, it is deemed acceptable given the improvement in the predicted dynamic field. At $Pr = 100$, the error between the Nu predicted by the LSMOD_YAPMOD model and the DNS data is significantly reduced from -59.1% to -1.3%. As such, the performance of the LSMOD_YAPMOD model in predicting thermal field for natural convection has indeed improved across a wide range of Prandtl number values.

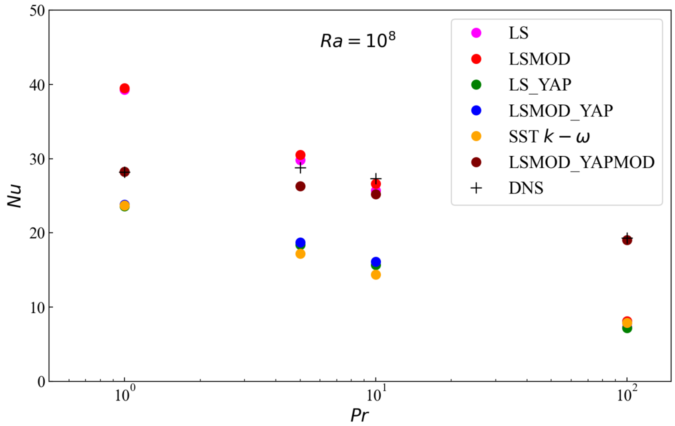


Figure 15: Nusselt number comparisons for natural convection channel flow over a range of Prandtl numbers $Pr = 1,\ 5,\ 10,\ 100$ at $Ra = 10^8$; Comparison between LS, LSMOD, LS_YAP, LSMOD_YAP, SST $k-\omega$, LSMOD_YAPMOD models and DNS data of Howland et al. [11].

5 Conclusion and Future Work

The primary objective of this research was to investigate the flow and heat transfer characteristics of high Prandtl number fluids using RANS methods across various test cases, and to improve RANS modelling for turbulent heat convection in these fluids. To achieve this, this study first assessed the performance of several standard and extended RANS models. Subsequently, we introduced and developed modifications to a widely used linear eddy-viscosity RANS model to enhance predictions of the thermal and dynamic fields. These modifications were then implemented into existing turbulence models within OpenFOAM and validated against DNS and experimental data.

For the forced convection channel flow cases, dynamic field predictions from the high-Re $k-\varepsilon$, SST $k-\omega$, and LSMOD models, agreed well with DNS data over the Re_τ range, while the LS and LRR models tended to underpredict turbulence within the log-law region. In the thermal field, a modified damping function term eddy-viscosity model was evaluated, extending to a broad range of Pr values. The results showed that the modified damping term increased eddy viscosity and turbulent heat flux near the wall in simple channel flows with uniform heat flux. At high Pr, where the thermal boundary layer was thin, the damping function tied to Re_τ effectively captured the thermal behaviour near the wall with minimal impact on the dynamic field.

For the natural convection channel flow cases, a turbulence model (LSMOD_YAPMOD) was proposed to improve predictions at high Pr, by modifying the Yap correction and introducing a modified damping function to enhance near-wall turbulent viscosity and heat flux. This model showed significant improvements in temperature predictions at $Pr = 5,\ 10$, with consistent Nu predictions across $Pr = 1,\ 5,\ 10$ when compared to the other LS variant models. At $Pr = 100$, the LSMOD_YAPMOD model significantly improved temperature and velocity predictions by enhancing turbulence near the wall.

In future work, three-dimensional time-dependent Rayleigh-Bénard convection flow and mixed convection channel flow involving high Prandtl number fluids will be evaluated using all the employed models, focusing particularly on the LSMOD_YAPMOD model proposed in this study. Some of the results for the three-dimensional time-dependent Rayleigh-Bénard convection flow are as follows;

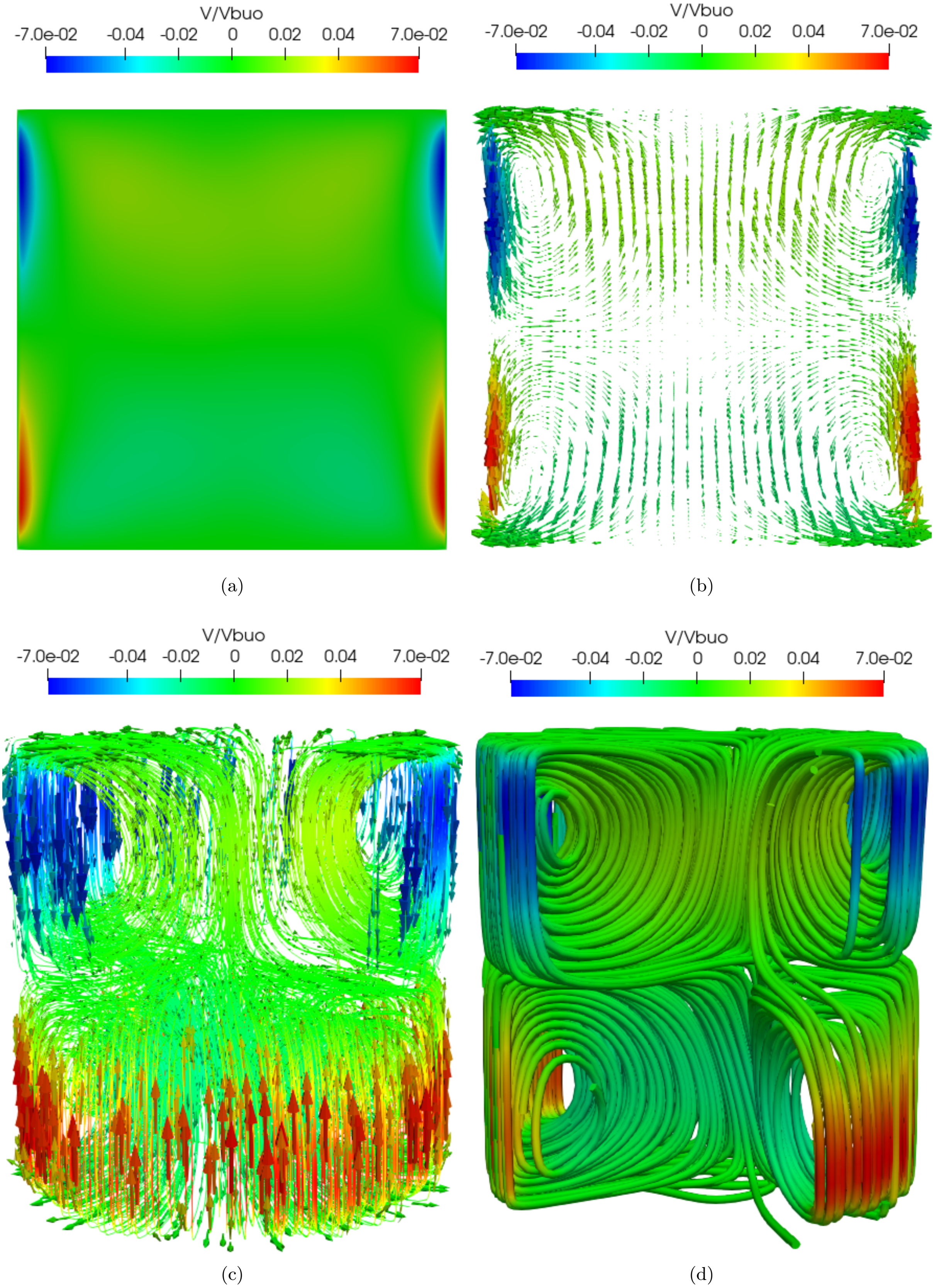


Figure 16: Non-dimensional time-averaged vertical velocity results for Rayleigh-Bénard convection.

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