Oral presentation | Numerical methods

Numerical methods-VII

Thu. Jul 18, 2024 2:00 PM - 4:00 PM Room A

[11-A-02] An immersed boundary wall-modeled large-eddy simulation approach for low and high-speed flows

*Christoph Brehm[1], Johan Larsson[1], William van Noordt[2] （1. University of Maryland, 2. University of Oxford）

Keywords: immersed boundary method, turbulent flow, numerical method

Immersed Boundary Wall-Modeled LES for Low- and High-Speed Flow Problems

C. Brehm[1], W. van Noordt[2], J. McQuaid[2], S. Ganju[1], & J. Larsson[1]

[1]*Department of Aerospace Engineering University of Maryland, USA*

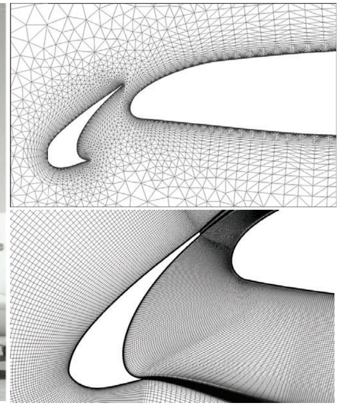
[2]*Whoosh HPC Lab, LLC, USA*

ICCFD12

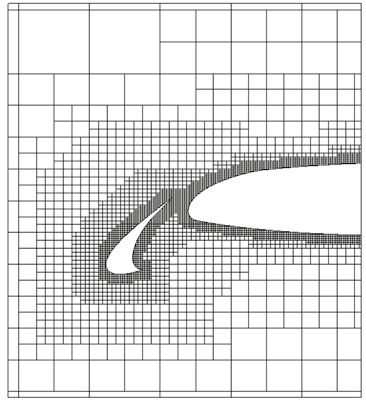
Grid Generation

"Real world" hardware.

State of-the-art meshes around an airfoil with deployed slat.



IBM can facilitate our ability to capture geometrically complex objects.



Outline

Brief Overview of Immersed Boundary Methods (IBM)

- Continuous Forcing, Cut-Cell Method, Ghost-Cell Method, & Sharp Finite Difference IBM

Some applications of IBM

- Rocket launch
- Contra-rotating open rotor
- Supersonic parachute inflation
- Wave packet tracking
- Droplet impact

IBM and Wall Modeled LES

- Numerical Implementation
- Some Validation Results + Performance

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Historic perspective of our journey to WM-IBM

Some applications of IBM

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IBM and Wall Modeled LES

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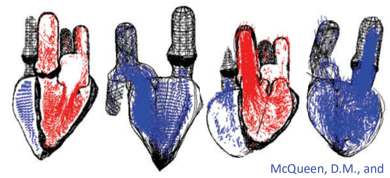
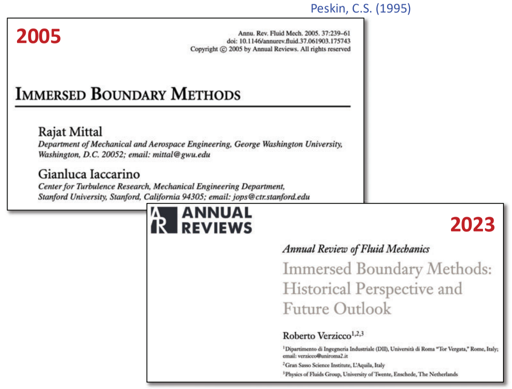
Different IBM Approaches

History of IBM

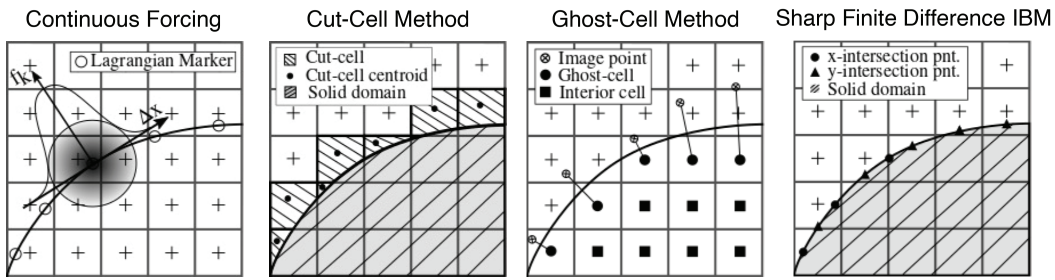
- ❑ Introduced in the 70s to simulate the flow in the human heart.
- ❑ Heart walls are modelled as elastic membranes.
- ❑ The interaction is modelled using a source term added to the governing equations. Peskin (1972,1977)
- ❑ IBMs appeared in many different forms:
  - Direct Forcing (Mohd-Yusof (1997), Goldstein et al. (1993))
  - Ghost-Cell Method (Fedkiw et al. (1999), Majumbar et al. (2001), Ferziger & Peric(1996), Iaccarino & Verzicco (2003), Mittal(2004)
  - Immersed Interface Method (Lee & Leveque (2003), Linnick & Fasel (2005)
  - Distributed Lagrange Multiplier (Glowinski et al. (1998), Projection Approach Taira & Colonius (2007))
  - Cut Stencil Method (Green et al. (2016), Duan et al. (2012))
  - Embedded Boundary Method (Balaras (2003), Yang & Balaras (2005), Collela et al. (2006), Schwartz et al. (2006), Barad et al. (2005)
  - Cut-Cell/Fluid Method (Clarke et al. (1986), Udaykumar et al. (2001,2001), Ye et al. (1999), Berger & Aftosmis (1998), Berger (1990), Meinke et al. (2013), Meyer et al. (2010), Melton et al. (1995), Örley et al. (2015), Pogorelov et al. (2015)
  - SBP+SATs (Mattson & Almquist (2016))
  - Discontinuous Galerkin (Müller et al. (2016))
  - *etc.*



McQueen, D.M., and Peskin, C.S. (1995)

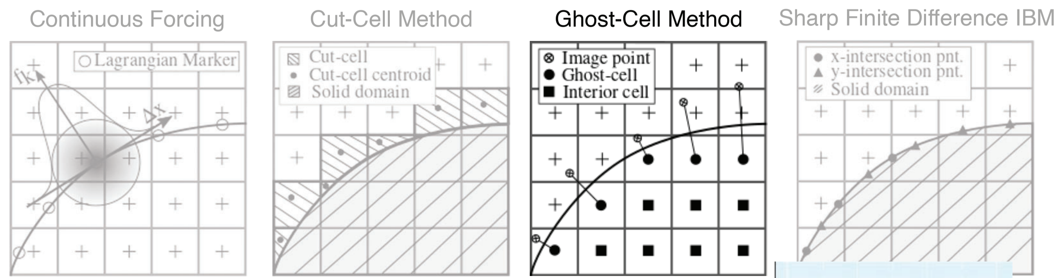


Different IBM Approaches

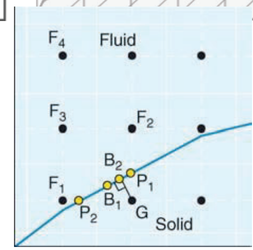


Boustani, Barad, Kiris, & Brehm (2021)

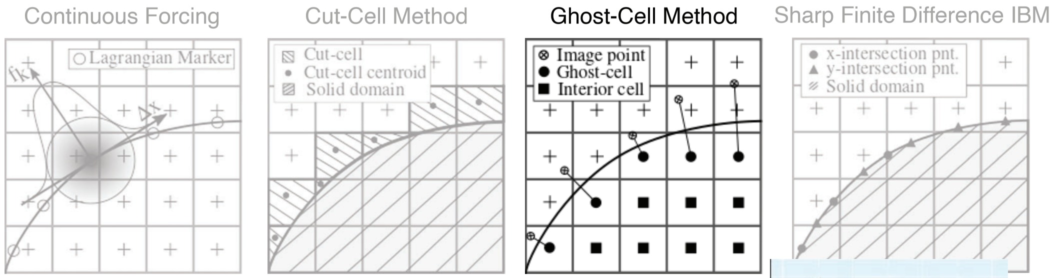
Different IBM Approaches



- ❑ Is the most popular IB approach because it is very simple to implement
- ❑ Many papers have been published on ghost-cell methods (Majumdar et al. (2001), Iaccarino & Verzicco (2003), Iaccarino et al. (2003), Mittal et al. (2004), Kalitzin et al. (2003), ….)
- ❑ Boundary condition on the IB is enforced through the use of “ghost cells”
- ❑ Interpolation scheme that implicitly incorporates the boundary condition on the IB is devised



Different IBM Approaches



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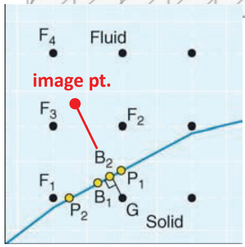
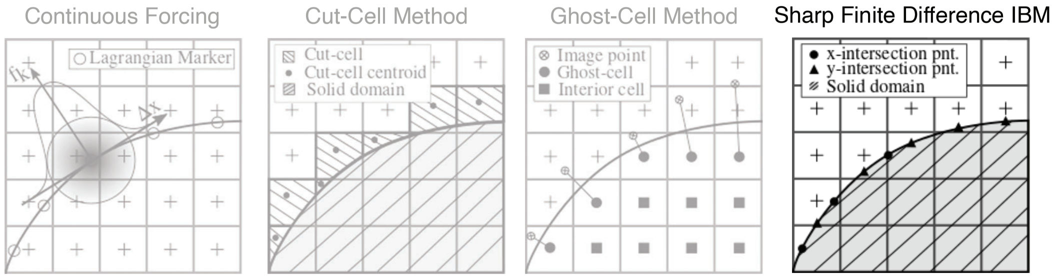
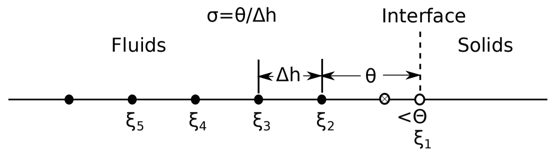


Different IBM Approaches



- ❑ Direct enforcement of boundary conditions at the wall
- ❑ Many different flavors have been developed
  - ❑ Jump corrected FD schemes
    Wiegmann & Bube (2000), Linnick & Fasel (2004), …
  - ❑ Use of irregular finite difference operators
    Brehm & Fasel (2013,2015), Duan et al. (2010), …



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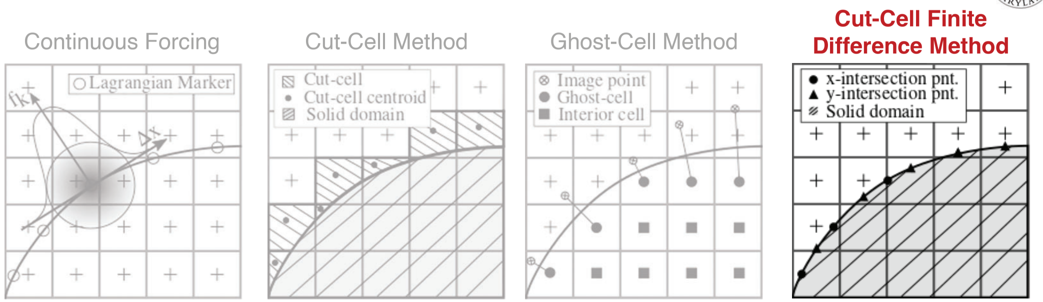

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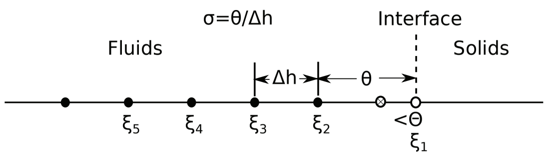
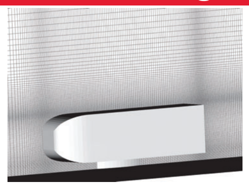
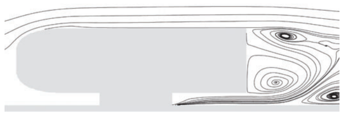


Early Applications of IBM

IBM Solutions Highlighted in Annual Review 2005

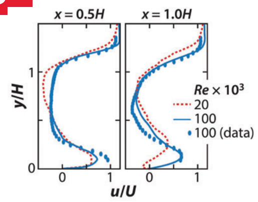


Non-uniform Cartesian mesh



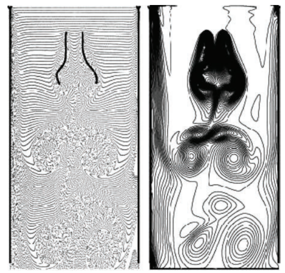
Khalighi *et al.*, 2001
Verzicco *et al.*, 2002

Streamtraces of mean flow at $Re = 20$

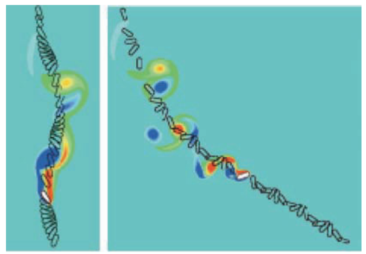


Vertical profiles of mean streamwise velocity

Flow around a 3D model road vehicle using the immersed boundary method



Clapping motions of two filaments in a flowing soap film Zhu., 2003



Simulation of a plate in free fall (cut-cell method) Mittal *et al.*, 2004

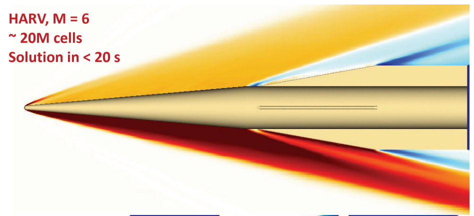
[1]Mittal, R., & Iaccarino, G. (2005). IMMERSED BOUNDARY METHODS. *Https://Doi-Org.Proxy-Um.Researchport.Umd.Edu/10.1146/Annurev.Fluid.37.061903.175743*, *37*, 239–261. https://doi.org/10.1146/ANNUREV.FLUID.37.061903.175743

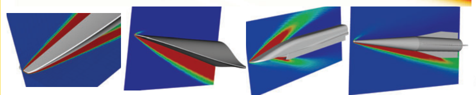
Hypersonic Vehicle Performance Analysis and Trajectory Simulations

***S**mall **B**usiness **I**nnovation **R**esearch Phase I/II Project:*

Digital Engineering - A Fully-Automated Simulation Approach for Hypersonic Vehicles

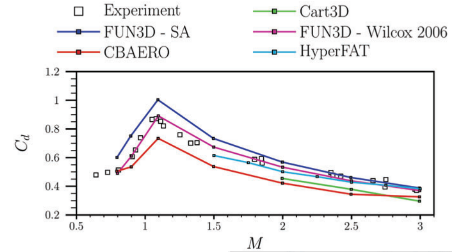
HARV, M = 6
~ 20M cells
Solution in < 20 s

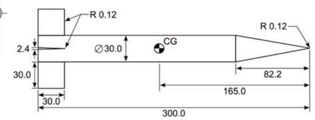




BOLT, M = 7 | **Waverider, M = 4** | **X-51, M = 4** | **HARV, M = 6**

Solutions for typical hypersonic vehicles can be obtained in < 1min (additional optimization possible)!





Solver	Required CPU-hours
FUN3D - Wilcox	56
FUN3D - SA	42
FUN3D - Euler	8
Cart3D	4
CBAERO	0.5
Present solver (serial, CPU)	1.5
Present solver (GPU)	0.0108

- Performance comparison for ANF geometry:
- GPU + FWSI dramatically outperforms other codes

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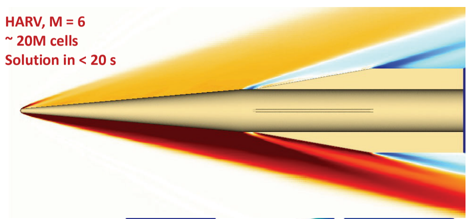

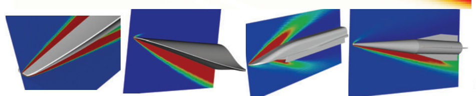

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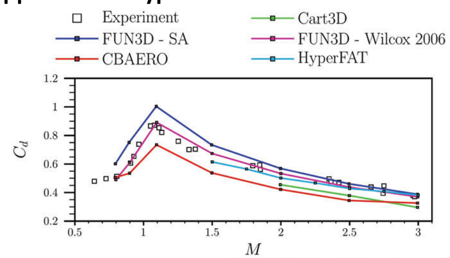

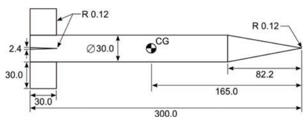

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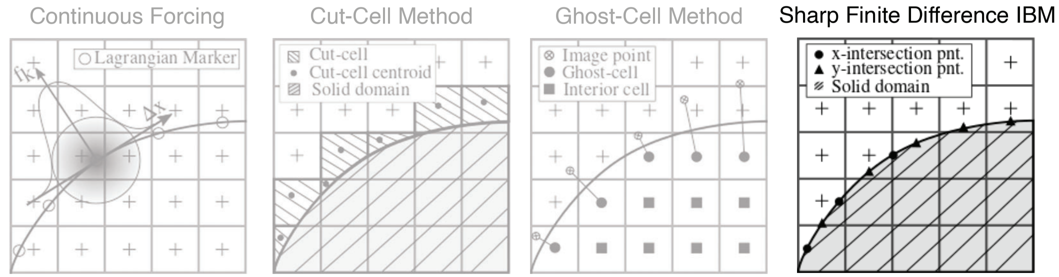
What was next?

Can we obtain truly higher-order IBMs?

Different IBM Approaches



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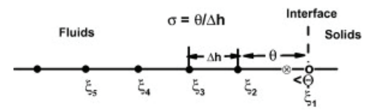
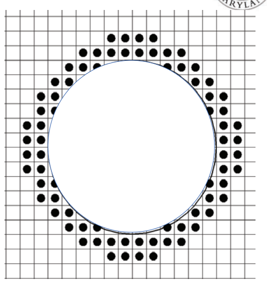


ICCFD12

Basic Idea of Stability Enhancement



- ❑ Initial observation:
  "Stability of numerical scheme can be formulated as N-dimensional optimization problem"
  (N=number of irregular grid points)
- ❑ Derivation of stencil coefficients:
  - Enforce order-of-accuracy

$$\tau_i = \left(-c_x\frac{\partial \phi}{\partial x} + \frac{\partial}{\partial x}\left(\beta\frac{\partial \phi}{\partial x}\right) + f\right)_{x=x_i} - \left(\hat{\phi}_B\tilde{\alpha}_1 + \hat{\phi}_i\tilde{\alpha}_2 + \hat{\phi}_{i+1}\tilde{\alpha}_3 + \hat{\phi}_{i+2}\tilde{\alpha}_4 + \hat{\phi}_{i+3}\tilde{\alpha}_5 + C\right)$$

  - Additional grid point is needed to introduce free parameter
  - Objective function depends on the nature of the PDE, e.g., $\lambda_{r,max}$ or $\rho(A)$

$$\frac{\partial \phi}{\partial t} = -c_x\frac{\partial \phi}{\partial x} + \frac{\partial}{\partial x}\left(\beta\frac{\partial \phi}{\partial x}\right) + f \xrightarrow{discr.} \underline{\mathbf{B}}\hat{\boldsymbol{\phi}}^{n+1} = \underline{\mathbf{A}}\hat{\boldsymbol{\phi}}^{n} + \hat{\mathbf{f}}$$

  - Extract perturbation of irregular finite difference stencil (assume **B=I**)

$$\underline{\mathbf{A}} = \underline{\mathbf{E}}\underline{\mathbf{A}}_{id} \xrightarrow{cond.} \rho(\underline{\mathbf{E}}) \leqslant 1$$

Brehm & Fasel (JCP, 2013, 2015), applications: Brehm, Barad, & Kiris (JCP 2018, IJFH 2017), Brehm, Barad, Housman & Kiris (JFM 2016)

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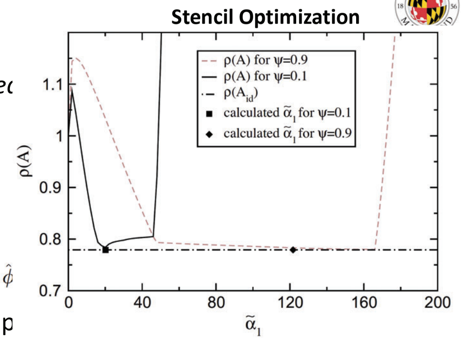
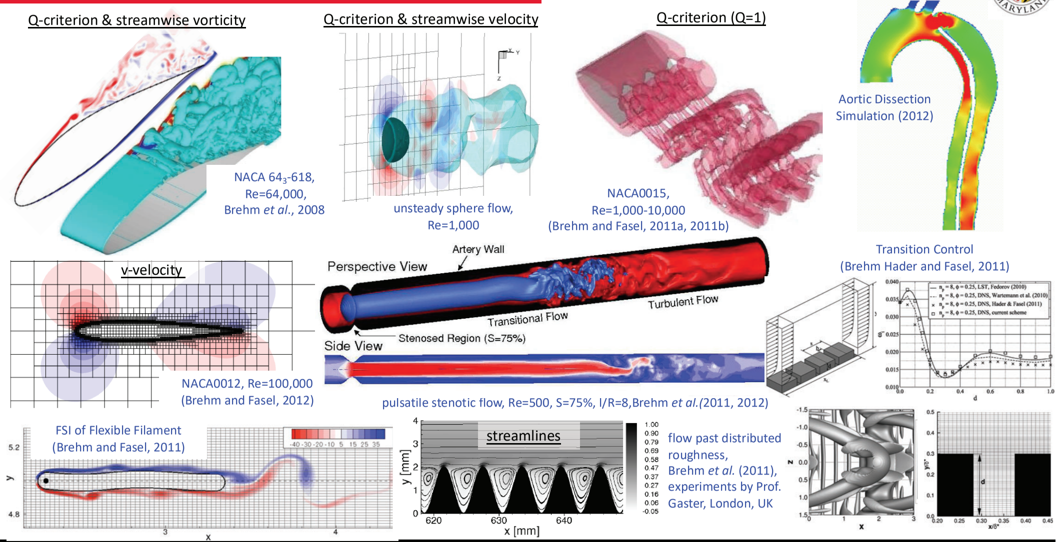


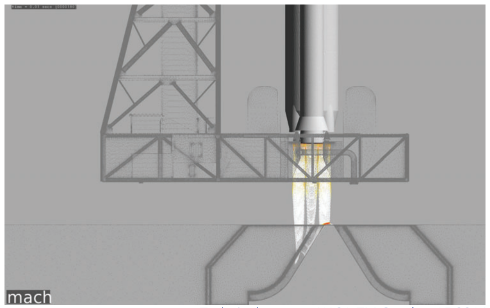
Brehm & Fasel (JCP, 2013, 2015), applications: Brehm, Barad, & Kiris (JCP 2018, IJFH 2017), Brehm, Barad, Housman & Kiris (JFM 2016)

Simple IBM Problems – Low Speed

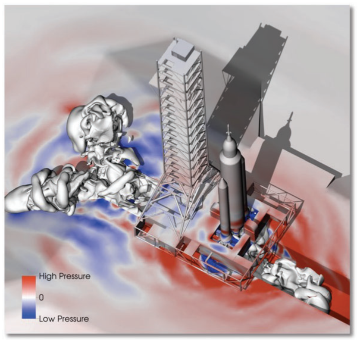


Can IBMs be used for practically relevant aerospace problems?

Rocket Launch Simulations



Technical reports: NASA-TM October 15 2014
ARC-E-DAA-TN7327 and ARC-E-DAA-TN10931



Kiris et al. (2015)

- ❑ Utilization of Immersed boundary method for design of launch infrastructure at NASA KSC
- ❑ Ignition over-pressure (IOP) analysis
- ❑ Multi-physics simulation (multi-species, multi-phase, etc.)
- ❑ Automatic volume grid generation from surface triangulation
- ❑ Adaptive mesh refinement (AMR) for tracking flow features with local refinement (gradient, entropy adjoint, and geometry based)

Courtesy of NASA Ames Research Center! See also https://www.nas.nasa.gov/SC22/research/project8.html for more recent simulations

Contra-Rotating Open Rotor

Low Speed (Take-Off)
(9 x 15 Low-Speed Wind Tunnel)

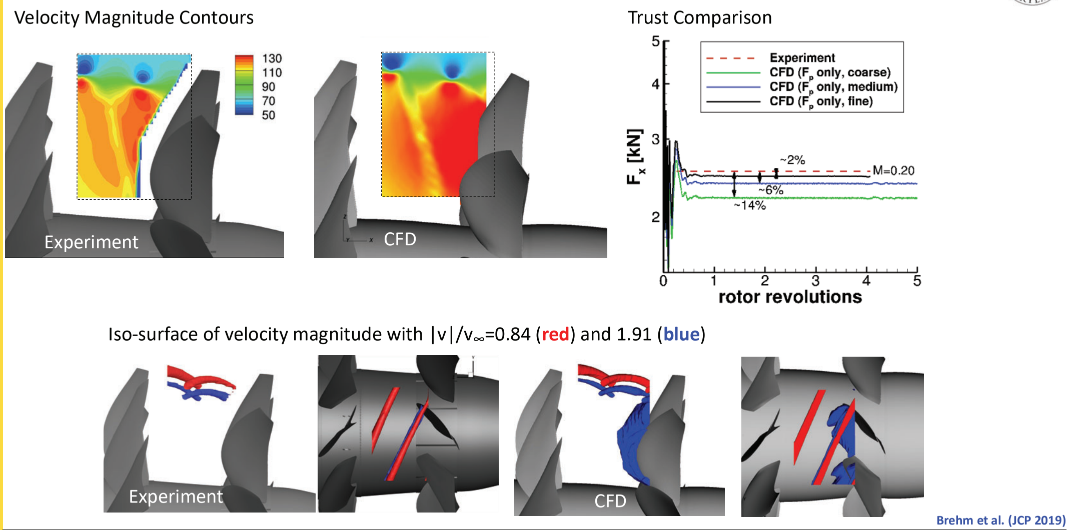
High Speed (Cruise)
(8 x 6 Supersonic Wind Tunnel)

Cases	Low Speed	High Speed
Rotation Speed [RPM]	6303/6303	6848/6848
Blade Setting (fwd/aft) [º]	40.1/40.8	64.4/61.8
Mach	0.20	0.78

Brehm et al. (JCP 2019)

Courtesy of NASA Ames Research Center! https://www.nas.nasa.gov/SC16

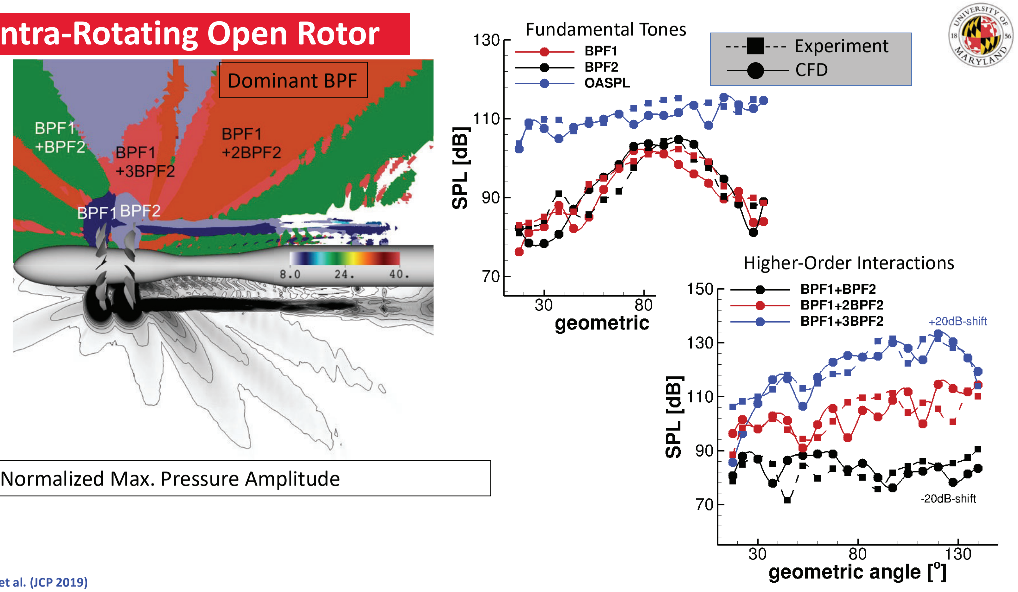
Contra-Rotating Open Rotor



Brehm et al. (JCP 2019)

Courtesy of NASA Ames Research Center!

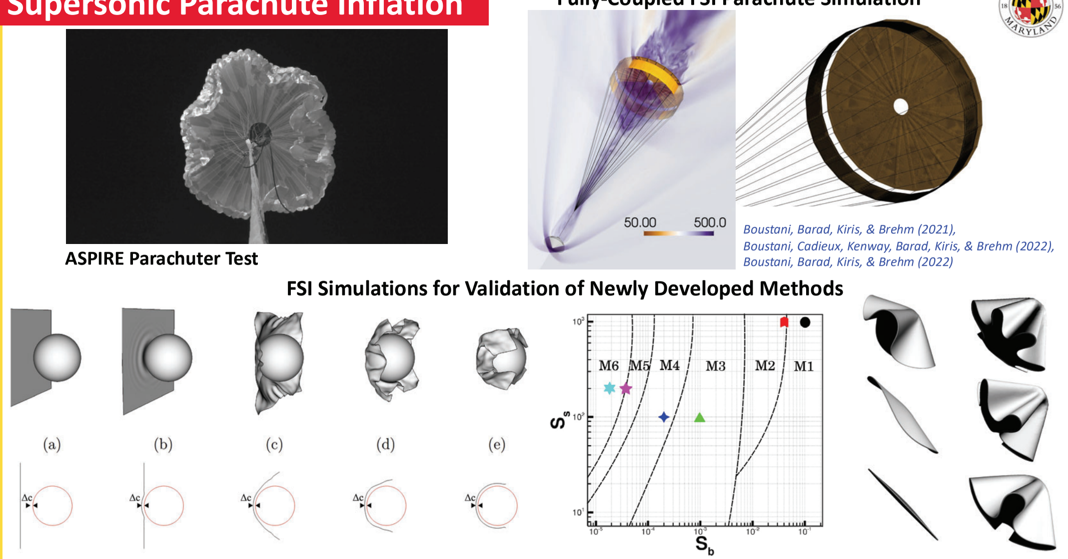
Contra-Rotating Open Rotor



Brehm et al. (JCP 2019)

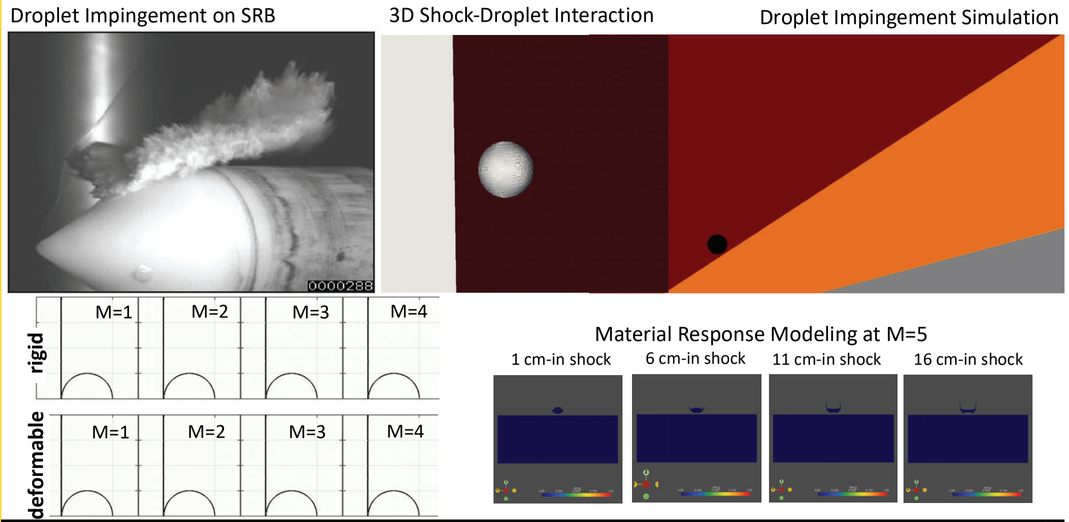
Courtesy of NASA Ames Research Center!

Supersonic Parachute Inflation



Boustani, Barad, Kiris, & Brehm (2021),
Boustani, Cadieux, Kenway, Barad, Kiris, & Brehm (2022),
Boustani, Barad, Kiris, & Brehm (2022)

Droplet Impingement



Application of IBM to RANS

... skipping this in the interest of time

Finally!
Application of IBM in context of WMLES

History of IBM

Annu. Rev. Fluid Mech. 2005. 37:239–61
doi: 10.1146/annurev.fluid.37.061903.175743

2005

IMMERSED BOUNDARY METHODS

Rajat Mittal
Department of Mechanical and Aerospace Engineering, George Washington University, Washington, D.C. 20052; email: mittal@gwu.edu

Gianluca Iaccarino
Center for Turbulence Research, Mechanical Engineering D... Stanford University, Stanford, California 94305; email: jops...

*[...] **High–Reynolds number flows constitute one of the main limitations** of IBMs owing to the resolution of thin wall shear layers, which cannot benefit from anisotropic grid refinement at the boundaries. To alleviate this weakness, researchers have developed [...]*

ANNUAL REVIEWS

2023

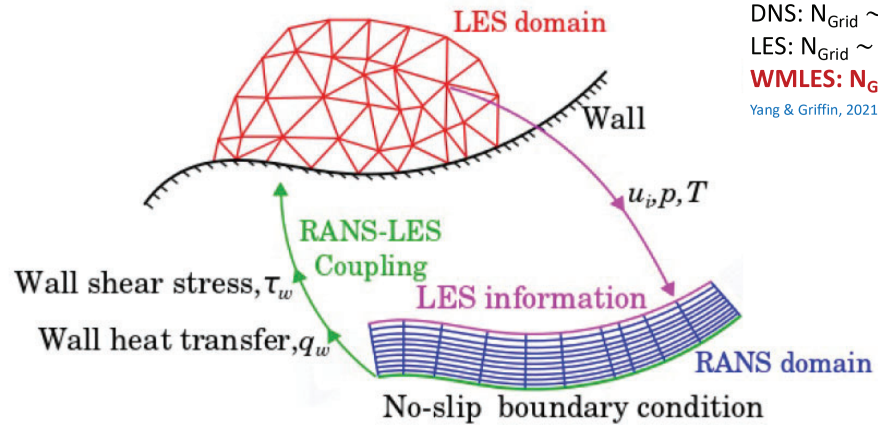
Annual Review of Fluid Mechanics

Immersed Boundary Methods: Historical Perspective and Future Outlook

Roberto Verzicco[1,2,3]

[1]Dipartimento di Ingegneria Industriale (DII), Università di Roma "Tor Vergata," Rome, Italy; email: verzicco@uniroma2.it
[2]Gran Sasso Science Institute, L'Aquila, Italy
[3]Physics of Fluids Group, University of Twente, Enschede, The Netherlands

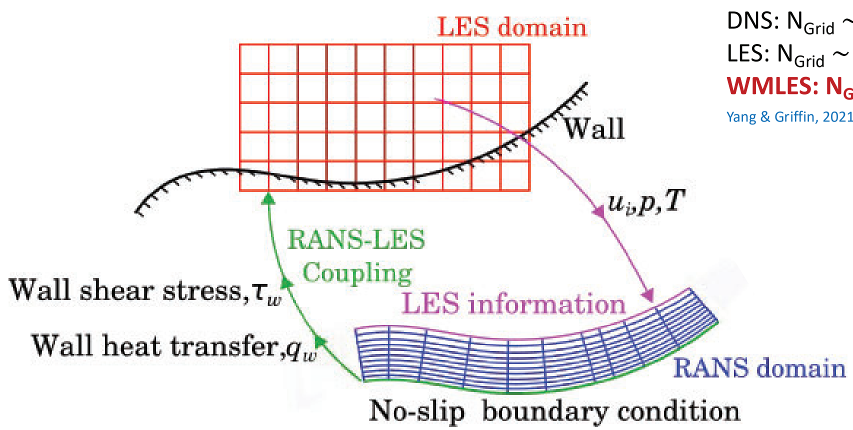
Wall Model Coupling to LES Domain



DNS: $N_{Grid} \sim Re_L^{2.91}$
LES: $N_{Grid} \sim Re_L^{2.72}$
WMLES: $N_{Grid} \sim Re_L^{1.14}$
Yang & Griffin, 2021

Adjusted from Bodart & Larsson (2011)

Wall Model Coupling to LES Domain



DNS: $N_{Grid} \sim Re_L^{2.91}$

LES: $N_{Grid} \sim Re_L^{2.72}$

WMLES: $N_{Grid} \sim Re_L^{1.14}$

Yang & Griffin, 2021

Adjusted from Bodart & Larsson (2011)

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The main objectives of this work are to assess:

1. *the effect of* ***boundary closures*** *used for IBMs when coupled with wall model*
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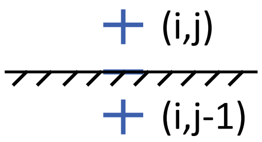

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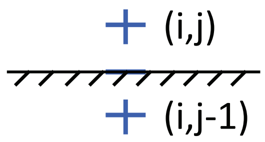

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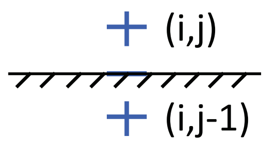

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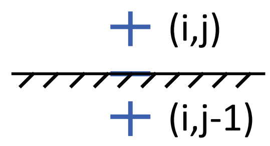

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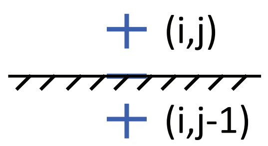

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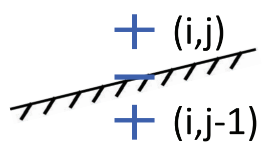

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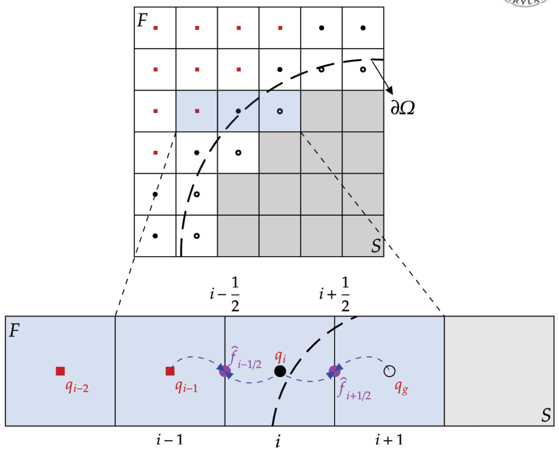
Numerics – Irregular Convective Terms

- Like regular treatment with order-reduction at IB
  - **4th** order kinetic energy and entropy preserving (KEEP) scheme[18,19]
  - stability and reduction of spurious noise
  - third-order upwinding for dissipation

$$\hat{f}^{(c)} = (1-\alpha)\hat{f}_{KEEP} + \alpha\,\hat{f}_{diss}$$

- Dissipation added using specially designed upwind operators[3]
  - One ghost used
  - Flux-vector splitting
  - Upwind biased
- Specialized procedures for ghost-cell filling

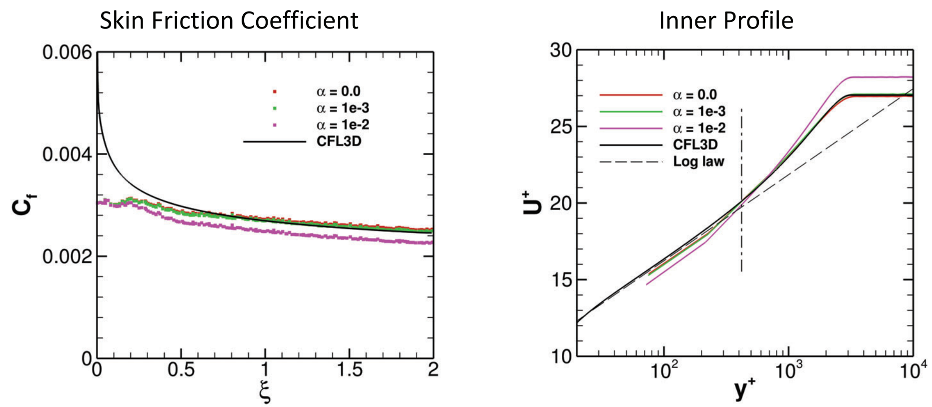


Flux reconstruction at the face for irregular KEEP operators

[3]van Noordt, W., **Ganju, S.**, & Brehm, C., JCP, 2022

Effects of Dissipation

Flat plate turbulent boundary layer flow (IBM-WM-RANS)



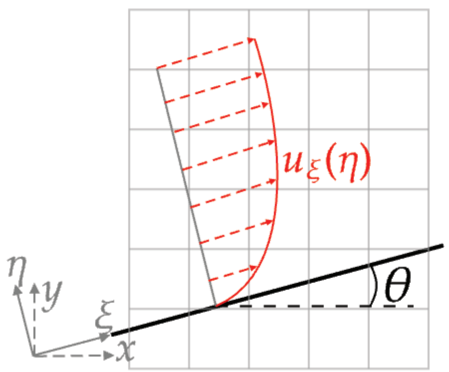
Ganju, van Noordt, & Brehm (submitted to JCP)

https://turbmodels.larc.nasa.gov/flatplate.html

ICCFD12

Numerics – Inviscid Tests

- Immersed boundaries can cause large errors for non-aligned Cartesian grids
  - Series of test functions used to assess the effect of IB operators on the solution
  - Flat plate geometry
  - Several angles of IB inclination (θ) tested
  - Simulations use **2nd-order KEEP** flux with no dissipation
- Test functions
  - Linear: $\vec{u_t} = a\eta + b$
  - Quadratic: $\vec{u_t} = a\eta^2 + b\eta + c$
  - Laminar profile (Blasius)
  - **Turbulent profile (Reichardt)**
- Test functions rotated into a local coordinate system (ξ, η) based on the local IB geometry
- Simulations are run until flow has settled



Schematic of the inviscid test functions used for validating the IB schemes

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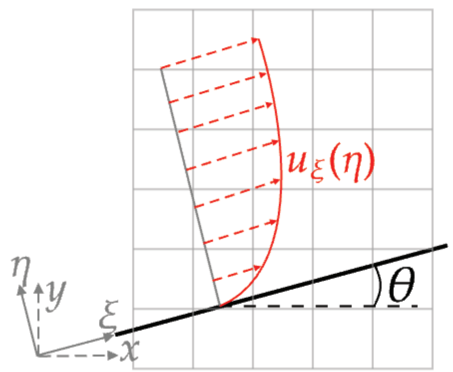

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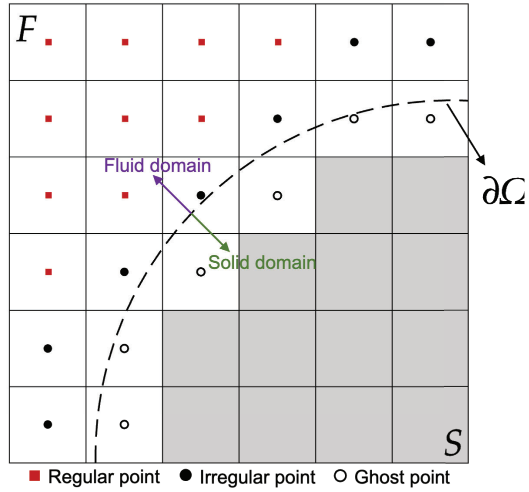
Inviscid Tests – Reichardt Profile

- Reichardt profile
  - Mimics a turbulent profile
  - Freestream adjusted to better approximate boundary layer profile
  - Very strong gradients near the wall
- $\vec{u}_{t,g} = c_1\vec{u}_{t,1} + c_2\,\vec{u}_{t,2}$
  - Extrapolate 1: $c_1 = 0, c_2 = 1$
  - Extrapolate 2: $c_1 = \frac{d_1}{d_1 - d_2}, c_2 = 1 - \frac{d_g}{d_1 - d_2}$



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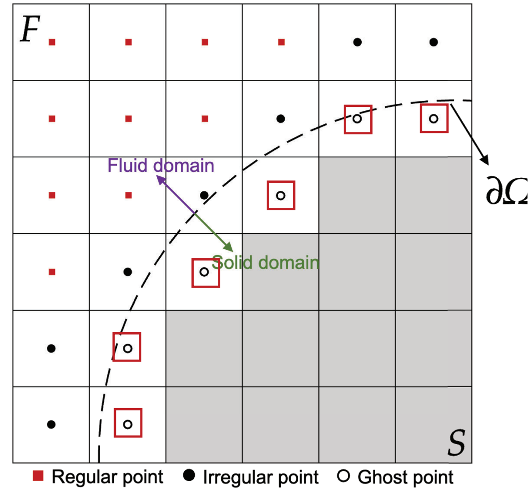

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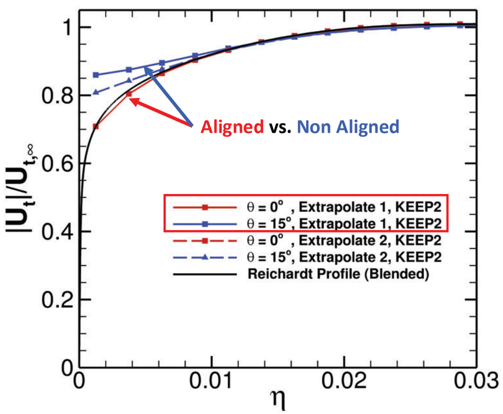

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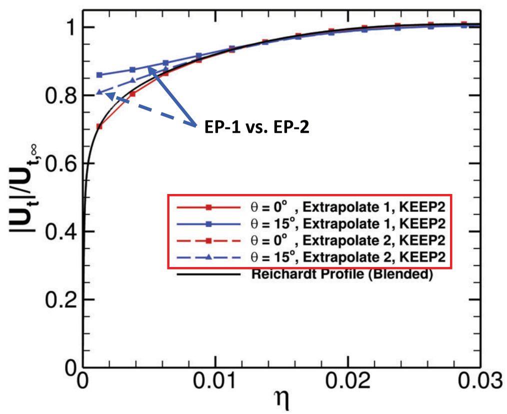

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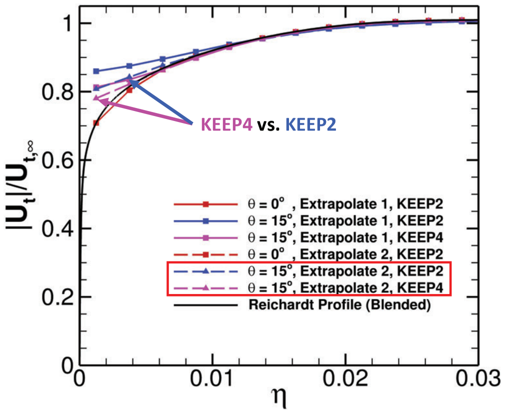

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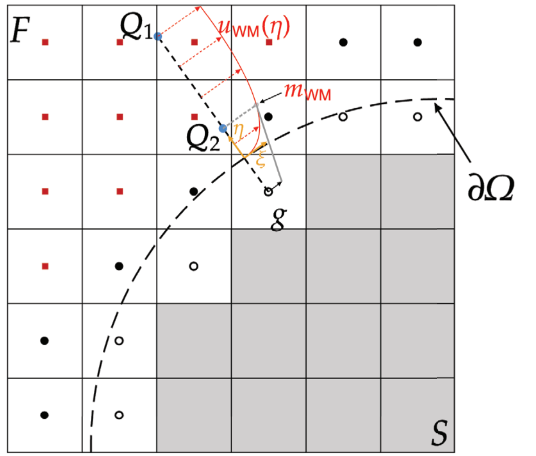
- Extrapolate WM
  - $\vec{u}_t = \vec{u}_{t,1} - m_{WM}(d_1 + d_g)$
  - Cartesian grid is severely under-resolved
  - Wall-model gradient is a better approximation of the **mean profile**



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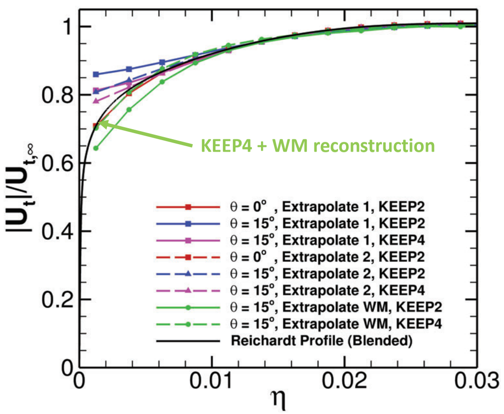

Ganju, van Noordt, & Brehm (submitted to JCP)

Inviscid Tests – Reichardt Profile

- Reichardt profile
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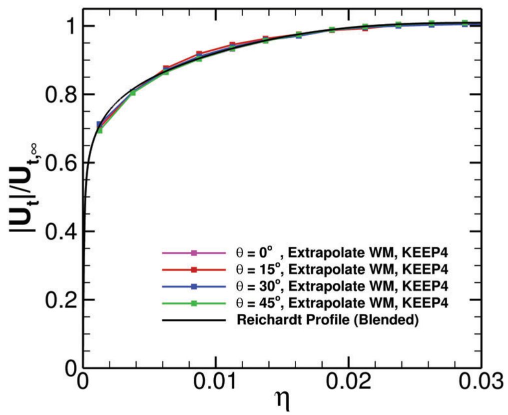
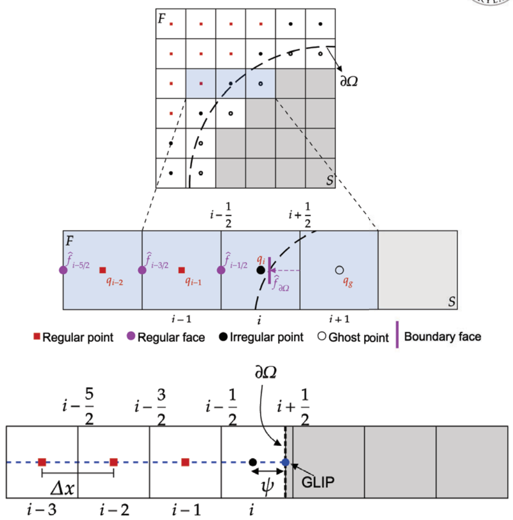
  - Freestream adjusted to better approximate boundary layer profile
  - Very strong gradients near the wall
- Varying domain inclination
  - Different angles of inclination tested
  - Coarse grid used
  - 4th-order KEEP scheme (regular)
  - Extrapolate WM BC



Ganju, van Noordt, & Brehm (submitted to JCP)

Numerics – Irregular Viscous Terms

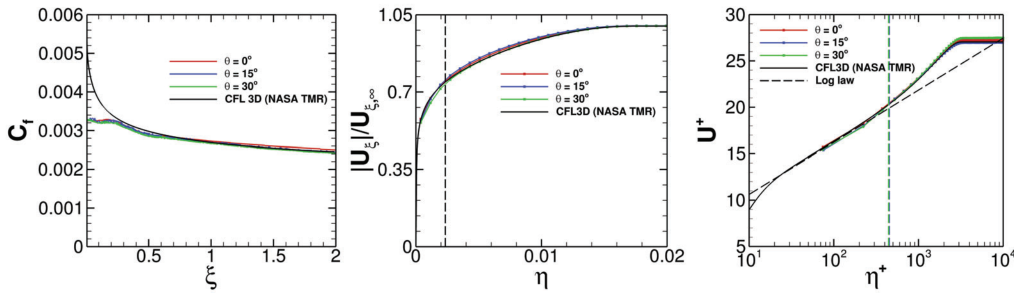
- Viscous fluxes are specified at grid-line intersection points
  - Intersection of the line formed by the cell-centers and the IB
  - $\hat{f}_{\partial\Omega}$ is obtained from the wall-model
  - Ghost-point is not used for evaluation of viscous fluxes
- Flux divergence at grid point i is evaluated as follows
  - Intersection of the line formed by the cell-centers and the IB
  - $\left.\frac{\partial \hat{f}}{\partial x}\right|_i = a\hat{f}_{\partial\Omega} + b\hat{f}_{i-1/2} + c\hat{f}_{i-3/2} + d\hat{f}_{i-5/2}$
  - 2nd-order flux divergence
  - Trying to mimic telescoping sum property



[3]van Noordt, W., **Ganju, S.**, & Brehm, C., JCP, 2022

Results – WMRANS of Turbulent Boundary Layer

- Re-visit the WMRANS turbulent layer case from NASA-TMR website
  - Numerical experiments show similar errors when using 2nd order KEEP operator in the regular cells
  - Extrapolate WM BC performs better than other BCs



Skin-friction coefficient — Mean streamwise velocity — Inner-scaled mean streamwise velocity

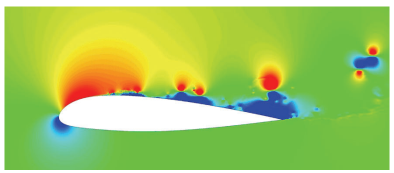
Ganju, van Noordt, & Brehm (submitted to JCP)

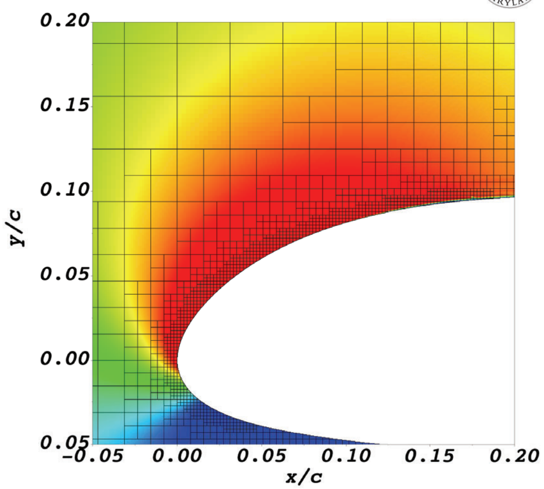
https://turbmodels.larc.nasa.gov/flatplate.html

Results – A-Airfoil

- Turbulent flow past A-Airfoil (recent AIAA paper)
- Flow features:
  - Non-alignment of grid and IB
  - Non-equilibrium flow effects – strong pressure gradients
- Part of the High-Fidelity CFD Verification Workshop[4]

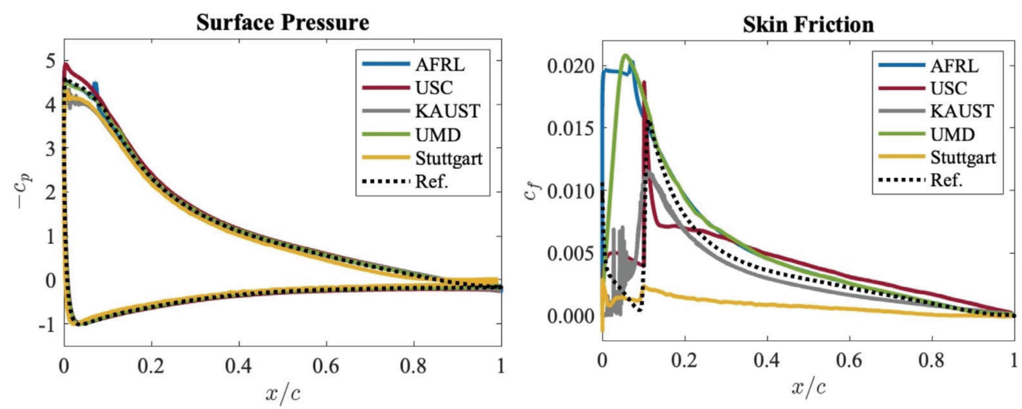
P_∞ (Pa)	T_∞ (K)	M_∞	Re_1 (m^{-1})	α
101327.0	300.0	0.15	10^7	13.3°





[4]Tamaki & Kawai, AIAAJ, 2023

WM Workshop Results – A-Airfoil



Garmann, Larsson, Brehm, et al. "Summary of the Wall-Modeled LES Test Suite from the 2024 AIAA High-Fidelity CFD Verification Workshop," AIAA Aviation 2024

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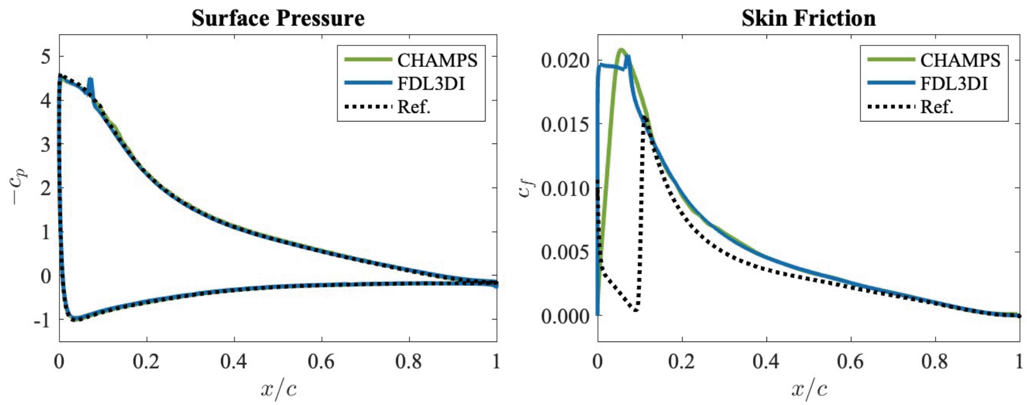

Thanks to Dr. Dan Garmann and Dr. Matthew Schwartz for sharing FDL3DI results!!!

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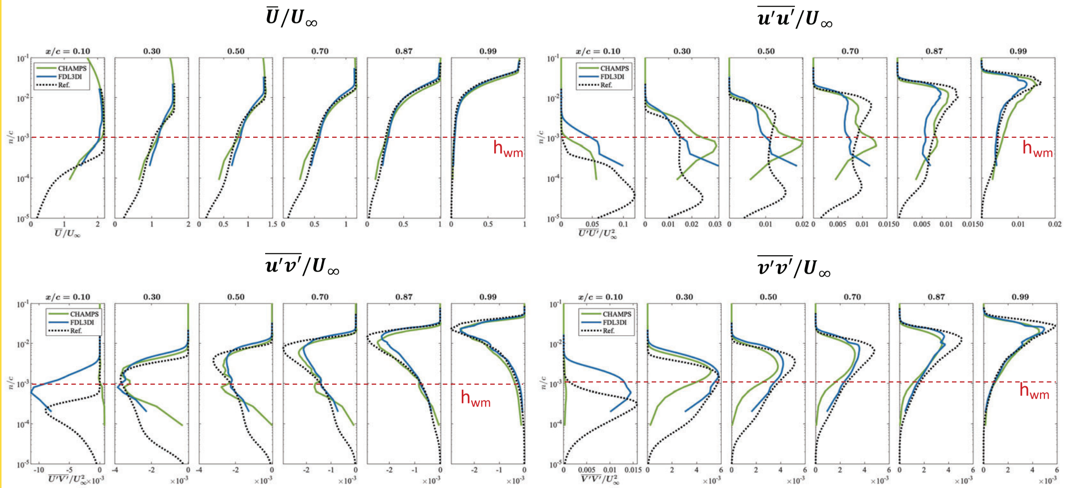
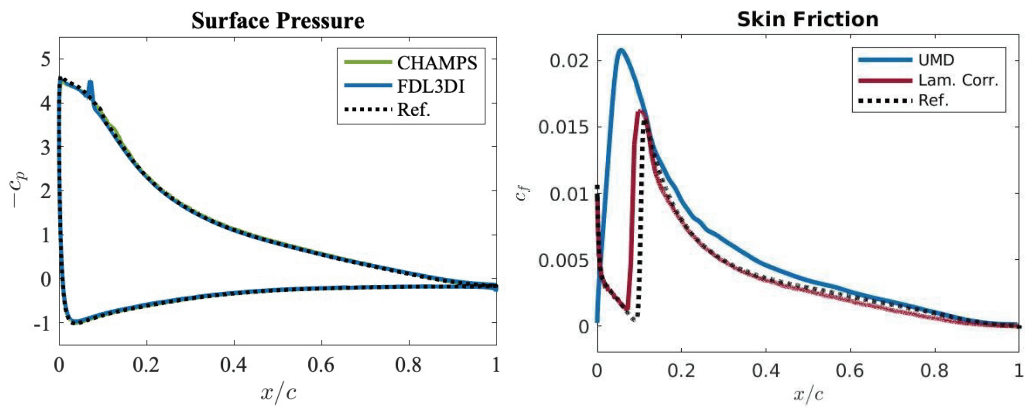


ICCFD12

Laminar BL treatment – A-Airfoil



Hussain, Ganju, Batstone, van Noordt, Larsson & Brehm, "Wall-modeled large eddy simulation of the A-airfoil using body-fitted and immersed-boundary codes," AIAA Aviation 2024

A Note on GPU Performance

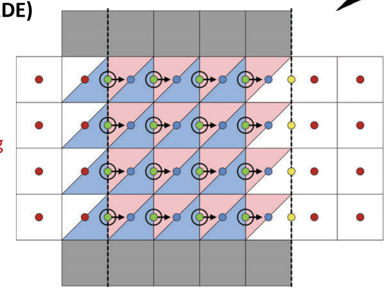
Full re-write of CHAMPS within C++20 using generic GPU backend (SPADE)

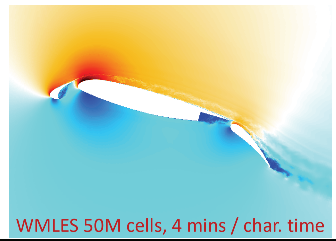
- Low-memory implementation allows for efficient GPU memory usage: approx 8-16M cells / GB of GPU memory
- All schemes, source terms, and boundary conditions written generically with no performance loss
- Compute bound GPU kernels, including for single-precision calculation
- Mixed-precision approach for handling geometry queries and sampling
- Anisotropic mesh refinement to allow high aspect ratio cells / domains

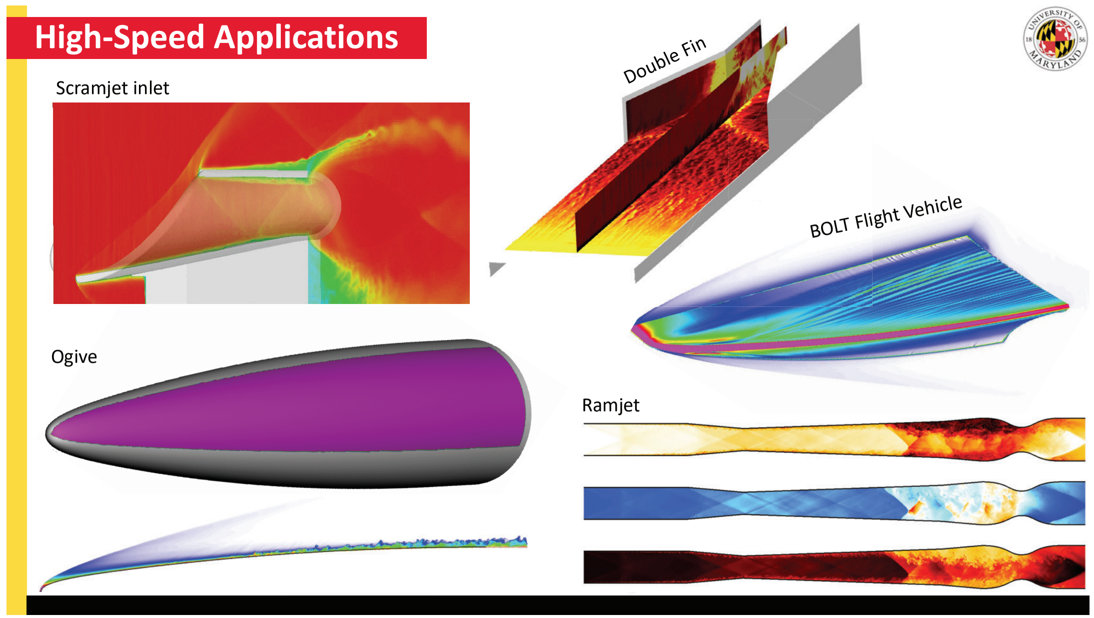
Potential for enabling rapid, high-fidelity analysis:

- 2x RTX 4090, 48GB = 400-500 million cells on a single workstation
- Depending on case complexity, 1.8-2.7 billion cells updated per second with 3-stage explicit scheme
- Approx. 140x faster than previous CPU implementation

Flux divergence algorithm utilizing optimal memory layout on GPUs






High-Speed Applications
Scramjet inlet
Double Fin
BOLT Flight Vehicle
Ogive
Ramjet