Oral presentation | Turbulence simulation (DNS,LES,RANS)

Turbulence simulation(DNS,LES,RANS)-III

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[10-B-01] Evaluation and improvement of turbulence model in supersonic rough-wall turbulent boundary layers

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Evaluation and improvement of turbulence model in supersonic rough-wall turbulent boundary layers

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1 Introduction

Surface roughness, that often arises on high-speed vehicle surfaces due to the application of material ablation for thermal protection techniques, significantly affects compressible turbulent boundary layers (TBL). However, previous studies predominantly concentrate on incompressible rough-wall TBL, yielding numerous significant findings that greatly facilitate the modeling of roughness effect. One of the most important conclusions is the log law in velocity profiles proposed by Nikuradse [1], which indicates that the logarithmic part of the velocity profiles still exists in rough-wall TBLs, but it generally shifts downward compared to the smooth cases due to the increase of wall stress. Based on this finding, Wilcox [2] modified the Reynolds-Averaged Navier-Stokes (RANS) model to predict the roughness effect without capturing the small-scale flow structures near the roughness. However, the Mach number effect and wall heat transfer effect on compressible rough-wall TBLs remain unclear and Wilcox's model needs to be further verified in compressible flows. In this paper, direct numerical simulations (DNS) are conducted to investigate the velocity profiles in compressible rough-wall TBLs. Then, the applicability of Wilcox's model in compressible rough-wall TBLs is discussed based on the DNS results. Finally, Wilcox's model is modified by the Reynolds analogy theory of rough walls.

2 Methodology

Navier-Stokes equation is solved numerically by a finite-difference method. Third-order Runge-Kutta method and seventh-order WENO-Z scheme are employed in DNS. The turbulence fluctuation library method [3] is adopted at the inflow boundary to construct a fully developed turbulent flow. Periodic boundary conditions are set at the spanwise boundary, and buffer zones are placed at the outlet of the streamwise and normal boundaries to prevent the interference from reflection disturbance. The distribution of roughness height for each roughness element can be expressed as follows:

$$y = 0.5k_c \sin\left(\frac{2\pi(x-x_0)}{x_r}\right)\sin\left(\frac{2\pi z}{z_r}\right) \tag{1}$$

where x_r and z_r are the wavelength in streamwise and spanwise directions, and k_c is the maximum peak-to-trough roughness height. In order to exclude the influences of the wavelength and roughness height, x_r/k_c and z_r/k_c are consistently set to 6 and the inner scaled roughness height (k_c^+) is about 80 in all cases. The equivalent sand-grain roughness height Reynold number (k_s^+) in all the cases is approximately 168. Flow conditions are designed as shown in Tab. 1, where Ma_∞ is the Mach number and T_w/T_{aw} is the ratio of wall-to-recovery temperature. The pressure and temperature of the free stream in all cases are 1240 pa and 64.4 K, respectively.

Table 1 Flow conditions for present DNS cases

Case	M2T1	M3T1	M5T1	M7T1	M7T2
Ma_∞	2.25	3.50	4.90	7.25	7.25
T_w/T_{aw}	0.843	0.843	0.843	0.843	0.432

In RANS simulation, the k-ω turbulence model is employed, and the boundary condition of ω is modified according to roughness modification proposed by Wilcox [2]. Moreover, the DPLUR method and Roe scheme are employed.

3 Results

3.1 Velocity profiles of DNS results

For the smooth cases, surface drag coefficient is only determined by friction drag coefficient (C_{fs}), while for rough cases, surface drag coefficient (C_{dr}) contains two components: friction drag coefficient (C_{fr}) and pressure drag coefficient (C_{dp}). C_{fr}/C_{fs}, C_{dp}/C_{fs} and C_{dr}/C_{fs} of all the cases are plotted in Fig. 1. Through the surface drag coefficient, the inner-scaled velocity u_τ can be calculated. The velocity profiles obtained by Van Driest velocity transformation of all the smooth and rough cases are plotted in Fig. 2. It is indicated that, the C_{dr}/C_{fs} is obviously influenced by the Ma_∞ and T_w/T_{aw}. Although an increase in Ma_∞ leads to an increase in C_{dr}/C_{fs}, the ΔU_{VD}^+ of M3T1 to M7T1 are almost the same. In regard to the wall heat transfer effect, a reduction in T_w/T_{aw} leads to a decrease in C_{dr}/C_{fs} and ΔU_{VD}^+ simultaneously. In summary, the log law still holds for compressible rough-wall TBLs, indicating that the theoretical basis of the Wilcox's model is still valid under compressible conditions.

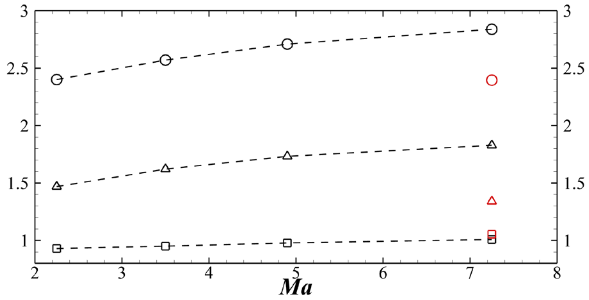

Figure 1 C_{fr}/C_{fs}(square symbol), C_{dp}/C_{fs}(triangle symbol) and C_{dr}/C_{fs}(circle symbol) of all cases, black symbols represent T1 cases, while red symbols represent T2 cases.

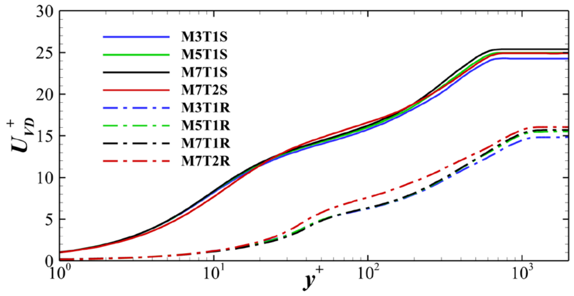

Figure 2 Van Driest Velocity profiles of smooth (solid line) and rough (dash-dotted line) cases under under M3T1 (blue line), M5T1(green line), M7T1 (black line) and M7T2 (red line) conditions.

3.2 Validation of present Wilcox's model

TBLs with the same flow conditions as the DNS cases are simulated by the RANS method for rough wall proposed by Wilcox. The drag coefficients and the ΔU^+ predicted by Wilcox's RANS method are plotted in Fig. 3 along with the DNS results. The C_{fr}/C_{fs} of RANS method doesn't exhibit the same Mach number effect as DNS data. Besides, the ΔU^+ decreases with the increase of Ma_∞, which deviates from the DNS data in sec. 3.1. It is indicated that, Wilcox's RANS model for rough wall requires corrections to be applied to super/hyper-sonic flows.

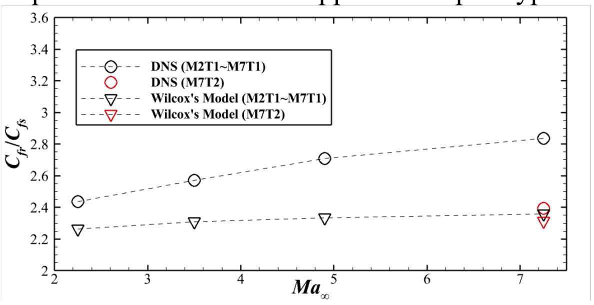

Figure 3 Comparison of C_{fr}/C_{fs} resules between Wilcox's RANS model and DNS data.

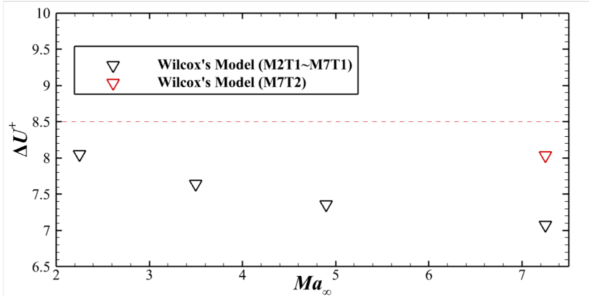

Figure 4 Comparison of ΔU^+ resules between Wilcox's RANS model and DNS data.

3.3 Modification of Wilcox's model

3.3.1 Mach number and wall heat transfer effect on C_f

To capture the Mach number and wall heat transfer effect in RANS model for rough surface, correction factor $\alpha > 1$ is introduced into the ω boundary conditions as follows:

$$\omega = \frac{u_\tau^2}{\nu} S_R, S_R = \begin{cases} \left(\frac{50}{\alpha k_s^+}\right)^2, k_s^+ < 25 \\ \frac{100}{\alpha k_s^+}, k_s^+ \geq 25 \end{cases} \tag{2}$$

The presence of α enhances the influence of k_s^+ compared to original model proposed by Wilcox. α can be determined by k_s^+ and ΔU^+. For fully rough wall ($k_s^+ > 70$),

$$\Delta U^+ = \frac{U_e}{u_{\tau,s}} - \frac{U_e}{u_{\tau,r}} = \frac{1}{\kappa}\ln(k_s^+) - 3.4 \tag{3}$$

After first order expansion, it can be rewritten as:

$$\frac{\Delta k_s^+}{k_s^+} = \frac{\kappa}{2}\Delta\left(\frac{C_{fr}}{C_{fs}}\right) / \left(\frac{C_{fr}}{C_{fs}}\right) * (U_{es}^+ - \Delta U^+) \tag{4}$$

The left side of the equation is α-1, and κ equals to 0.41. U_{es}^{+}-ΔU^{+} can be estimated by roughness function, and the result is $28.4 - 1/\kappa\ ln(k_s^+)$. $\Delta\left(\frac{C_{fr}}{C_{fs}}\right)/\left(\frac{C_{fr}}{C_{fs}}\right)$ is the changes in roughness-induced drag increment caused by Mach number and wall temperature effects. Based on the DNS data under M2T1~M7T1 conditions, a local quantity is defined, $Ma_\tau = \frac{u_\tau}{a_w}$,to measure the Mach number effect on $\Delta\left(\frac{C_{fr}}{C_{fs}}\right)/\left(\frac{C_{fr}}{C_{fs}}\right)$ as shown in Fig. 5. The fitting equation is:

$$\Delta\left(\frac{C_{fr}}{C_{fs}}\right)\Big/\left(\frac{C_{fr}}{C_{fs}}\right) = 25Ma_\tau^{2.7} \quad (5)$$

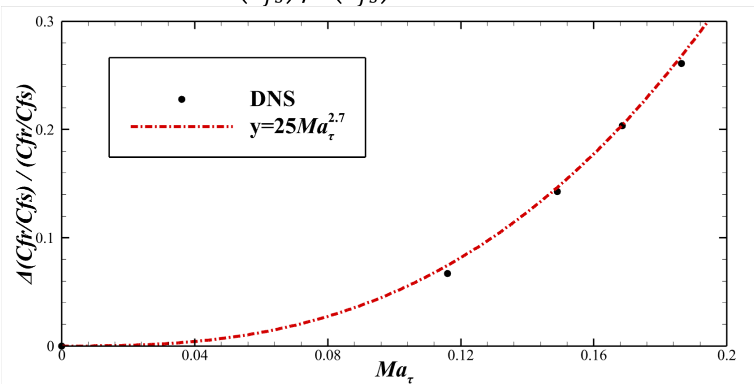

Figure 5 $\Delta\left(\frac{C_{fr}}{C_{fs}}\right)\Big/\left(\frac{C_{fr}}{C_{fs}}\right)$ as a function of Ma_τ

However, when the Mach number remains constant but the wall temperature decreases, the equation (5) needs to be corrected by another local quantity, $\frac{T_\tau}{T_w} = \frac{Q_w}{\rho_w C_p u_\tau T_w}$. Then, $\Delta\left(\frac{C_{fr}}{C_{fs}}\right)/\left(\frac{C_{fr}}{C_{fs}}\right)$ is represented as:

$$\Delta\left(\frac{C_{fr}}{C_{fs}}\right)\Big/\left(\frac{C_{fr}}{C_{fs}}\right) = 25Ma_\tau^{2.7} * \max\left(\frac{T_\tau/T_w}{0.009*(1+f)}, 1.0\right)^{-1} \quad (6)$$

Finally, α can be expressed as follows:

$$\alpha = 1 + \frac{1}{\kappa} * 25 * Ma_\tau^{2.7} * \max\left(\frac{T_\tau/T_w}{0.009*(1+f)}, 1.0\right)^{-1} * \left(28.4 - \frac{1}{\kappa} * \ln(k_s^+)\right) \quad (7)$$

3.3.2 Improving the prediction of wall heat flux

The equivalent roughness method usually leads to the over prediction of wall heat flux Q_w. B. Aupoix[4] modified the turbulent Prandtl number ΔPr_t to improved heat transfer predictions on rough surfaces:

$$\Delta Pr_t = (A\Delta U^{+2} + B\Delta U^+)\exp\left(-\frac{y}{h}\right)$$
$$A = (0.0155 - 0.0035S_{corr})(1 - \exp[-12 * (S_{corr} - 1)]) \quad (8)$$
$$B = -0.08 + 0.25\exp[-10 * (S_{corr} - 1)]$$

The turbulent heat transfer coefficient is thus reduced and the predicted Q_w is more accurate. But for super/hyper-sonic flows, Reynold stress work plays a considerable role in the energy transfer process. The work done by Reynold stress also needs to be reduced to overcome the problem of Q_w over prediction in super/hyper-sonic flows. Referring to the method of Aupoix[4], the correction factor β is

$$\beta = \frac{Pr_{t,smooth}}{Pr_{t,smooth} + \Delta Pr_t} \quad (9)$$

In this way, by reducing turbulent heat conduction and work done by turbulent stress simultaneously, the prediction of wall heat flux can be improved in super/hyper-sonic flows.

3.3.3 Validation of new model

Firstly, the predicted C_{fr}/C_{fs} by new model and DNS data are compared as shown in Figure 6 and Figure 7. It shows that the predicted roughness-induced drag increment is quite accurate. And Q_{wr}/Q_{ws} of M7T2 by new RANS model is 1.60, which is quite close to the DNS result of 1.63.

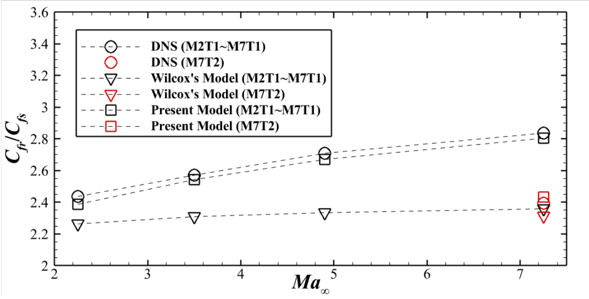

Figure 6 Comparison of C_{fr}/C_{fs} resules between Wilcox's RANS model, new model and DNS data

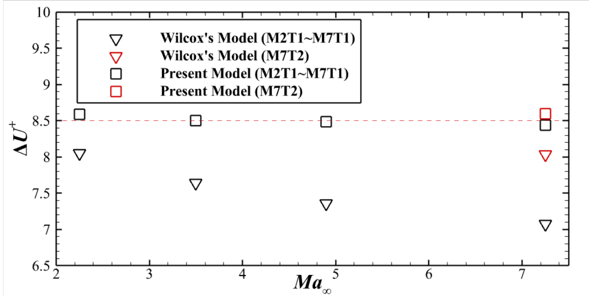

Figure 7 Comparison of ΔU^+ resules between Wilcox's RANS model, new model and DNS data

Next, flows over sharp cone with smooth surface and rough surface are simulated. The Q_{ws} and Q_{wr} predicted by present RANS model are compared with the experimental results of Holden[5].As shown in Figure 9, the results are in good agreement, verifying the accuracy of present model.

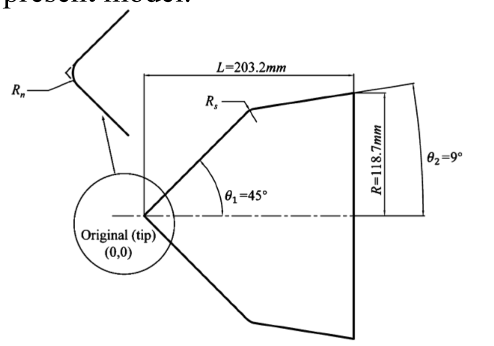

Figure 8 The sharp cone model in Holden's experiment

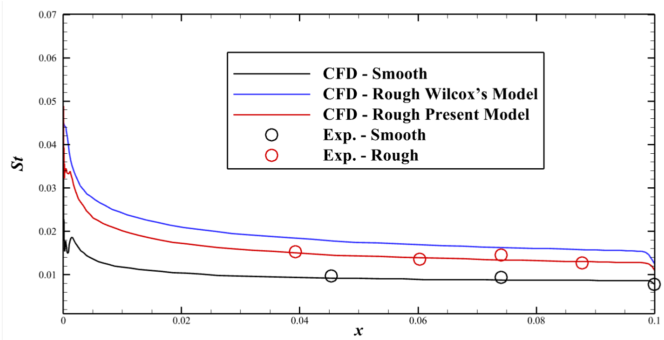

Figure 9 Comparison of St between RANS results and experimental results.

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