Obstacle Location and Identification using Time Reversal and Deep Learning

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Abstract: Finding unknown obstacles in a medium using numerical methods is an important task, with many applications. It is also very challenging, since the properties of the obstacle often have many degrees of freedom. Most existing methods utilize prior knowledge of the obstacles they aim to recover, and use optimizers that take long time to extract the obstacle location, shape and size. We propose a method based on time reversal and deep learning, for locating obstacles and finding their shape and size. We test the method on synthetic data that was generated to mimic the physical experiment of acoustic waves propagating in an underwater medium. In addition, we use a physically-informed neural network to get even more accurate results.

Keywords: Deep Learning, Time reversal, Wave propagation, Obstacle identification.

1 Introduction

In this work we present a new method, based on Deep-Learning (DL), for locating and identifying obstacles in an underwater acoustic domain. This problem has been widely investigated in the scientific community [1, 2, 3, 4], since it has many applications [5, 6, 7]. In a physical experiment, an acoustic wave propagates in the sea, and the acoustic pressures generated by this propagating wave are recorded by a small set of sensors. These recorded pressures are the data available for analysis. The two popular tasks are: a) given recorded pressures and knowledge about the medium, locate the source that initiated the wave propagation [7], and b) given recorded pressures and knowledge about the sources that initiated the wave propagation, find properties of the medium. In this paper we focus on the latter, where the property of the medium we are interested in, is a small reflecting scatterer placed inside the medium. To conduct this analysis we simulate data of the forward acoustic wave propagation. We use a numerical method to compute the forward propagation process, and save the computed pressures only at a small set of points selected inside the grid. We aim to find the location, shape and size of the scatterer based on the synthetic measurements in the sensors.

The problem of locating scatterers in a underwater acoustic domain based on a set of sensor measurements is an inverse problem [8, 9], and so highly ill-posed. Because the number of sensors is much smaller than the computational grid, we expect the solution to be highly sensitive to the data. A solution does not necessarily exist and if so, may not be unique. In addition, small changes in the conditions of the problem may lead to large changes in the solution. Moreover, the scatterer is usually of arbitrary shape, and to represent many different scatterers numerically we need many degrees of freedom. The partial information and complex representation make the problem of reconstructing the scatterer even more challenging.

We simulate a physical experiment, also called the forward process. We place a source somewhere in the computation grid. This source emits an acoustic wave that propagates throughout the medium from time 0 to time T. We choose a small set of coordinates on the grid and call them "sensors". We save only the data simulated on these points, but for every time step. We have only this data for the inverse problem analysis. An example of a single sensor recording is shown in Figure 1.



Figure 1: An example of sensors recording over time.



Figure 2: An illustration of the reconstruction of a source using TR for two different circular obstacles in the medium. Notice that the background is entirely different in the two reconstructions, which is holding some information about the obstacle.

Machine-Learning (ML), and specifically DL, are hot topics in the scientific community. With the recent advances in computational hardware, enabling high scale computations, the field erupted with many researchers creating data-driven solutions for their problems [10, 11, 12, 13]. One improvement is Physics-Informed Neural Networks (PINNs) [1], that are a family of DL models "physically aware" of the PDE of which they are trying to approximate the solution. PINNs have shown robustness, ability to fit non-linear problems, fast convergence (compared to optimization problems), simple to design and implement, and many other advantages. PINNs are used mostly for forward problems, but can be used for inverse problems as well.

A disadvantage of ML methods in general, PINNs included, is when facing a problem with too many degrees of freedom, training the networks may not converge. In the obstacle location and identification problem, different obstacles create a very large sample space, so one needs to train a neural network to infer the scatterer location, shape and size (which vary a lot) from a small data-set (sensor recordings). This is very challenging for both numerical methods and ML methods. A method for solving this problem was proposed in [11]. In this work, a spatio-temporal architecture was proposed to shift the information from the sensors recordings into an image segmentation of the obstacle. In addition, a physics-informed loss term was introduced, to achieve even better performance.

We propose an innovative method that incorporates the Time-Reversal (TR) [14, 15] method into the solution proposed in [11]. The TR method propagates the acoustic waves backwards through time, until a reconstruction of the initial condition is achieved. Due to the partial information, this reconstruction is not perfect. In addition, having a scatterer inside the domain gives us a different reconstruction for every different scatterer. An example of two different reconstructions is shown in Figure 2. These small differences are the information we are going to use to extract the location, shape and size of the obstacle. More precisely, the influence of the scatterer on the sensors measurements can be reflected in the different backgrounds of the initial condition reconstructions. We propose a data-driven method that can extract the location, shape and size of the scatterer from these reconstructions.

We first create a data-set of arbitrary scatterers (varying location, shape and size). We then use a numerical solver to solve the forward problem and create a synthetic data-set for solving the backward

problem. We emphasize that we have information about the obstacle only when solving the forward part (which mimics a physical experiment, where the obstacle lays in the domain). When solving the backward problem we have no knowledge about the obstacle. The proposed method for solving the inverse problem consists of three blocks. We first apply the TR process to the sensors data to produce, for each arbitrary source, a reconstruction of the initial condition. We then use a deep neural network to extract the image of the source from this data. The third part involves a physically-informed loss term that utilizes the knowledge of the wave problem, leading to even better performance. We also compare the success of the proposed method to the results in [11].

2 Numerical formulation of the problem

2.1 Mathematical formulation of the physical problem

The wave problem is given by

$$\begin{cases} \ddot{u}(\vec{x},t) = \nabla \cdot (c^2(\vec{x})\nabla u(\vec{x},t)) & \vec{x} \in \Omega, \quad t \in (0,T] \\ u(\vec{x},0) = u_0(\vec{x}) & \vec{x} \in \Omega \\ \dot{u}(\vec{x},0) = v_0(\vec{x}) & \vec{x} \in \Omega \\ u(\vec{x},t) = f(\vec{x},t) & \vec{x} \in \partial\Omega_1, \quad t \in [0,T] \\ \frac{\partial u}{\partial \mathbf{n}}(\vec{x},t) = g(\vec{x},t) & \vec{x} \in \partial\Omega_2, \quad t \in [0,T] \end{cases}$$
(1)

where $u(\vec{x}, t)$ is the wave amplitude or acoustic wave pressure, and $c(\vec{x})$ is the wave propagation speed (assumed constant throughout this paper). The initial conditions are given by u_0, v_0 for the acoustic wave pressure and velocity, respectively. The boundary conditions are given by f, g for the Dirichlet and Neumann boundary condition types, respectively. Throughout this paper, Ω is considered a two dimensional rectangular domain, and the initial conditions u_0, v_0 are compactly supported in Ω . In addition, throughout the paper we discuss only Dirichlet boundary condition, so $\partial \Omega = \partial \Omega_1$. The specific conditions (choice of initial and boundary conditions, domain, parameters, etc.) are defined in section 4.

This is a general formulation of the wave problem. We note that the wave problem is well-posed for given initial conditions (u_0, v_0) and boundary conditions (f, g), so there exists a unique solution, and small changes of the initial condition results in small changes of the solution. Solving the forward problem means finding the solution $u(\vec{x}, t)$ that satisfies the problem (1).

2.2 Numerical scheme for the forward problem

To create the synthetic data we approximate the solution of the forward acoustic wave propagation problem using Finite Differences (FD). We define a grid of size $N_x \times N_y$, equally spaces with Δx and Δy as the lengths between the nodes in the x and y axes respectively. The source is placed, throughout the paper unless stated otherwise, at the grid node coordinate $(\lfloor \frac{N_x}{4} \rfloor, \lfloor \frac{N_y}{4} \rfloor)$. The sensors are grid points $(x_k, u_k)_{k=1}^{N_s ensors}$, and the number of sensors $N_{sensors}$ is much smaller than the number of grid nodes $N_{sensors} \ll N_x N_y$ (usually 5-10 sensors in an experiment).

To approximate the solution we use the Finite Differences Central Differences (FDCD) marching scheme. We use a simplified notation and write:

 $u_{i,j}^n \approx u(x_{min} + i\Delta x, y_{min} + j\Delta y, n\Delta t, \quad i = 0, ..., N_x, \quad j = 0, ..., N_y, \quad n = 0, ..., N_t. \ x_{min} \text{ and } y_{min} \text{ are the lower bounds of the } x, y \text{ axes, respectively. We approximate the derivatives of the equation } \ddot{u} = c^2(u_{xx} + u_{yy}), \text{ assuming a constant velocity } c \text{ on the entire grid, using FDCD}$

$$\frac{u_{i,j}^{n+1} - 2u_{i,j}^n + u_{i,j}^{n-1}}{\Delta t^2} = c^2 \left(\frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{\Delta x^2} + \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{\Delta y^2} \right).$$
(2)

Some methods, e.g. the Newmark method, also use the velocity $\dot{u}(\vec{x},t)$ for computation, and some even use the acceleration $\ddot{u}(\vec{x},t)$. We utilize only on the pressures both for the forward and the inverse problems. For the forward process we solve for u^{n+1} while for the backward process we solve for u^{n-1} . To create the data we solve the forward process, and later when we use TR in section 3.1 we solve the backward process with



Figure 3: An illustration of the setup we use for most of the numerical experiments throughout this work. The source is a compactly supported thin Gaussian. The sensors are grid 8 grid points. An arbitrary polygon is shown here as an example.

the same FDCD approach. This FDCD scheme is an explicit compact second order accurate scheme in both space and time. The time step is limited by the Courant-Friedrichs-Lewy condition $\Delta t \leq \frac{1}{c\sqrt{2\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)}}$.

We use this to forward propagate the waves through N_{steps} time steps, and "record" the pressures at the sensors, resulting in a matrix of size $N_{steps} \times N_{sensors}$, containing the synthetically generated acoustic pressure measurements.

2.3 Formulation as a data-driven problem

We formulate the obstacle identification as a data-driven problem. Following [11], we use image segmentation techniques to find and identify the obstacles. We create binary images where pixels with the value 1 are inside the obstacle, and pixels with the value 0 are in the background. To randomly generate the obstacles, we create arbitrary polygons by generating a random number of edges, random edge lengths, and random angles between the edges. The number of edges and edge lengths were generated using a normal distribution while the angles were generated using a uniform distribution. We created a total of $N_{samples}$ arbitrary polygons, for the obstacles data which we denote as $\{\mathcal{O}_m\}_{m=1}^{N_{samples}}$. We also call these labels.

We use an initial source of a compactly supported thin Gaussian. An illustration of the setup including the source, the locations of the sensors and an arbitrary polygonal obstacle is given in Figure 3. Using FDCD to compute the forward propagation problem, we compute the data in the sensors. The obstacle is set in the domain by setting the wave propagation velocity in the domain to $c^2(1 - \mathcal{O}_m)$ (for each $m = 1, ..., N_{samples}$). Inside the obstacle the wave propagation speed is set to 0 (no propagation), and outside it the speed is c^2 . Then, we generate another arbitrary polygon and solve the forward problem again with a different obstacle image. This is done for all $N_{samples}$. We eventually achieve a three dimensional matrix of size $N_{samples} \times N_{steps} \times N_{sensors}$, with corresponding binary images of obstacles used to simulate the forward process, a three dimensional matrix of size $N_{samples} \times N_x \times N_y$. The data-driven problem is then to fit a set of sensor recordings $\{u_m(t_n, x_k, y_k)\}_{\substack{n=1,k=1\\n=1,k=1}}^{N_{steps},N_{sensors}}$ for each sample $m = 1, ..., N_{samples}$, to a corresponding binary image of the obstacle $\{\mathcal{O}_m\}_{m=1}^{N_{samples}}$.

3 Deep learning framework

We propose using a method composed of three steps:

- 1. Preprocessing and feature extraction using TR.
- 2. Convolutional layers that predict the location and shape of the obstacle.
- 3. Physics-informed loss term used only during training.

The input to the method is the sensor recordings. We first apply TR to get the reconstruction of the source. Then, we train a convolutional neural network that learns to extract the obstacle from the image. Last, we introduce a physics-informed loss term that utilizes the wave equation, to create another loss term for training the network. The additional loss term acts as a penalty term, yielding better training and overall better performance of the method. We now discuss each step and the contribution to the overall method.

3.1 TR based feature extraction

In this step we use FDCD, as discussed in section 2.2, to backward propagate the waves using the sensor recordings. In the backward step we do not have any information about the obstacle. To use the FDCD, we first initialize the grid with 0 values. We place the values recorded in the sensors in the corresponding coordinates of the grid at the final time step $(u_{i,j}^{N_{steps}})$, and at the time step before it $(u_{i,j}^{N_{steps}-1})$. We then compute $u_{i,j}^{N_{steps}-2}$ and substitute the computed values at the sensors coordinates with the recorded values there from the forward step, at that time. We do this iteratively using (2) until we get $u_{i,j}^{0}$, which is the desired reconstruction.

Hence, this step transforms each sample from size $N_{steps} \times N_{sensors}$ to a matrix of size $N_x \times N_y$. Note that in relevant applications $N_{sensors} \ll N_{steps}$, while $N_x \approx N_y$, so we start with an uneven rectangular-like input and get a square-like output. One advantage of this transformation is that we immediately get the desired image shape $N_x \times N_y$ which allows us to use encoder-decoder (image to image) architectures. We achieve this in a mathematically meaningful way, without using any upsampling or interpolation methods.

Furthermore, this process performs a type of feature extraction. As seen in figure 4, its result is a sparselike image with higher values concentrated around the source, and smaller values approaching zero farther away from the source. By switching to a different representation of the data we enable the network to learn from new rich features that could improve the performance of the model.



Figure 4: An example of the TR step. On the left-hand side we have one sample taken from the raw dataset. It is composed of multiple sensor recordings over time. On the right-hand side we can see the output of the TR preprocessing step, which is a source reconstruction in the form of a single image.

3.2 Convolutional architecture

This step involves a deep neural network, trained to predict the obstacle images. During training, we use Stochastic Gradient Descent (SGD) to find the best set of parameters of the network that minimizes a predefined loss function. We use the Dice loss function [16, 17] defined by:

$$Dice := \frac{1}{N_{samples}N_xN_y} \sum_{m=1}^{N_{samples}} \frac{2\langle \mathcal{O}_m, \tilde{\mathcal{O}}_m \rangle_F}{\|\mathcal{O}_m\|_F^2 + \|\tilde{\mathcal{O}}_m\|_F^2},\tag{3}$$

where $\{\mathcal{O}_m\}_{m=1}^{N_{samples}}$, $\{\tilde{\mathcal{O}}_m\}_{m=1}^{N_{samples}}$ are respectively the sets of true and predicted labels, and $\langle \cdot, \cdot \rangle_F$, $\|\cdot\|_F^2$ are the Frobenius norm. Note that the predicted labels $\{\tilde{\mathcal{O}}_m\}_{m=1}^{N_{samples}}$ are not binary - they are probability images. For numerical stability, a small $0 < \epsilon$ value is often added to the numerator and denominator. The Dice loss is a differentiable metric for the comparison between the true and predicted images.

To extract the obstacle from the reconstruction of the initial condition made by TR in the previous step, we employ a variant of the DeepLabv3+ [18] architecture. This architecture uses an encoder-decoder structure, where the input and output of the neural network have the same shape. It was designed for the semantic segmentation problem from computer vision, which is the task of assigning pixel-level labels for an image. DeepLabv3+ has been shown to achieve state-of-the-art (SOTA) results on benchmark semantic segmentation datasets such as PASCAL VOC 2012 [19]. Due to the similarity of the obstacle problem to semantic segmentation, using a SOTA architecture from this domain is a natural choice.

DeepLabv3+ requires a backbone neural network to form the basis of the architecture. In the original paper, the authors used the Xception [20] architecture. In this paper we use a lighter version of Xception. We reduce the number of convolutional filters in each layer of the middle flow part of the original architecture by a factor of 4. The full modified DeepLabv3+ architecture ends up having around four million trainable parameters, which is a relatively small amount in terms of modern deep learning.

3.3 Physics-informed loss term

In addition to the Dice loss defined above, we add another loss term to the training procedure. This term is based on the FDCD described in 2, and is unique to the wave problem, since FDCD is a numerical solver of the wave equation. This makes the network aware of the wave equation, and the method is uniquely tailored for the wave problem data. This method can be extended to other problems as well, using an appropriate solver of the desired problem.

During the training iterations, every sample goes through the network to produce a prediction, which is a predicted image of the obstacle. We then use the forward FDCD for the predicted image (exactly how it was used to create the data in section 2.3), and compute the sensor recordings from the forward process. Unlike the true obstacle image, the predicted one is not binary, but is a probability image. The method is appropriate, since the term $c^2(1 - \tilde{O})$ produces velocities close to 0 inside the obstacle and close to c^2 elsewhere (assuming the network trains correctly). Therefore, we produce comparable sensor recordings (true versus predicted). We compare the sensor recordings to the true sensor recordings, which are the inputs to the TR block. We then compare the true sensor recordings to the predicted ones using the Mean Squared Error (MSE) defined by

$$PI = \frac{1}{N_{samples}N_{steps}N_{sensors}} \sum_{m=1}^{N_{samples}} \sum_{n=1}^{N_x} \sum_{k=1}^{N_y} \left| u_m(t_n, x_k, y_k) - \tilde{u}_m(t_n, x_k, y_k) \right|^2.$$
(4)

If the network trained perfectly (the prediction and the truth are identical), the same forward solver would have been applied to the same obstacle image, producing the same sensor recordings and the physics-informed loss would be 0. However, this is an impossible scenario. The better the network learns, the lower the physics-informed loss. Since the FDCD solver used to compute the forward process is linear (operates by summations and multiplications only), the automatic differentiation mechanism in Tensorflow 2 [21] is able to compute the gradients and no additional implementation is needed.

To train, we take a combination of the Dice loss and the physics-informed loss. We train the network and print the values of the Dice loss and the physics informed loss. We found that the two losses are of the

same order of magnitude. Thus, the loss we use for the training is the sum of the two losses. We recommend checking this before training, because if, for a different setup, one discovers large differences between the values, then one may consider weighting the two terms of the loss, giving a larger weight to one of them to bridge the gap. Failing to do so may make the smaller loss term redundant and insignificant, leading to saturation (the network would not converge).

4 Numerical tests and results

We created 60,000 samples as described in section 2.3. We chose a grid of size 128×128 with a physical size $[0, \pi] \times [0, \pi]$. We used 8 fixed sensors as shown in Figure 3. We emphasize that randomly generating the obstacles, as described in section 2.3, creates an enormous number of possible samples, and we are using only 60,000. The total storage size of the dataset is 8.1GB. The parameters we used are $c = 1484\frac{m}{s}$ (average acoustic wave propagation speed in the Mediterranean sea), $\Delta x = \Delta y = \frac{\pi}{127} \approx 0.025$, and $\Delta t = \frac{1}{c\sqrt{2}\left(\frac{1}{\Delta x^2}\frac{1}{\Delta y^2}\right)} \approx 8.33 \cdot 10^{-6}$. The total number of time steps is 500, so that the wave-front impacts the

boundaries and reflects back to the domain. A larger number of time steps means more interactions inside the domain, resulting in more data recorded in the sensors. However, the memory consumption when using more time steps is higher. Choosing 500 steps was based on a balance between enough recorded data and less computational needs. The initial condition is a small compactly supported Gaussian of the form $Ae^{\frac{-((x-x_0)^2+(y-y_0)^2)}{2\sigma}}$, with A = 1 and $\sigma = 1$. The boundary conditions are of homogeneous Dirichlet type

Are 2^{σ} , with A = 1 and $\sigma = 1$. The boundary conditions are of homogeneous Dirichlet type (reflecting).

To train the network we used 54,000 (90%) of the samples as the training set and the remaining 6,000 (10%) as the validation set. We created an additional 15,000 samples for testing the trained model on samples that are completely new. During training, we observed the loss values of the training set and the loss values of the validation set through the training iterations (also called epochs). When both losses declined, the network was training. When the validation loss stopped decreasing, the network is saturating, and when the validation loss started increasing, the network was over-fitting the training data. We saved the model with the lowest validation loss (before saturation or over-fit). We used 400 epochs and a batch size of 32 for training. We also tuned the learning rate of the ADAM [22] optimizer. We chose an initial learning rate of 10^{-3} , and decreased it by half whenever the loss plateaued. We trained the network on a nVidia A6000 GPU for 8 hours. After training, inference takes a fraction of a second (applicable for real-time purposes). An example output of the model, compared to the true obstacles, is shown in Figure 5. From example 1 we observe that the method is able to infer very thin obstacles. From example 2 we see that some of the predictions are very accurate in terms of Intersection Over Union (IOU, defined below). From example 3 we see that the method performs well also for obstacles near the boundary. From example 4 we see that nonconvex obstacles have been generated and used in training and testing (this is a test-set example), showing that this method is not constrained by convex obstacles. Example 5 highlights a phenomenon that exists in all other examples as well - we see how the model struggles with pointy shapes, and tries to infer a more smooth edged obstacle. This can be improved with tuning the hyper-parameters of the network, but the current results are satisfactory.

To evaluate the success of the method we used the IOU metric: $IOU = \frac{|\mathcal{O} \cap \tilde{\mathcal{O}}|}{|\mathcal{O} \cup \tilde{\mathcal{O}}|}$, for some sample where

the true obstacle image is \mathcal{O} and the prediction is \mathcal{O} . We first applied a threshold of 0.5 on \mathcal{O} , since it is a probability image. We used this metric to evaluate the predictions on the testing set by checking the mean, median and standard deviation of the IOU over the entire test set. We compared the results to the results in [11], which used a similar approach but did not use the TR method. The results are given in Table 1.

To check the contribution of the PI loss term, we conducted an experiment with 5,000 samples and compared a model trained with the PI loss and one without PI loss. We monitored the validation IOU score during the training and the results clearly indicate that the PI loss network converges faster. However, running the model with the PI loss was computationally heavy and time consuming, so we managed to show the potential on the small data-set. The other experiments were trained without the PI loss.

We also analyzed the influence of adding more training data improve the accuracy of the model. As the results in Table 1 clearly indicate, increasing the size of the training data-set produced a more accurate



(e) Obstacle identification example 5

Figure 5: Visual examples of outputs of the obstacle identification for different obstacles. Left is the true obstacle image, and right is the network inference output

model. To conduct these experiments we created data-sets consisting of 10,000, 20,000, 30,000 and 60,000 samples, split into 90% training and 10% validation. In addition, we observed that even for the 20,000 samples data-set we already achieved similar results to the benchmark [11].

5 Conclusion

We presented a method for finding the location, shape and size of an underwater obstacle. We used a finite difference scheme to approximate the solution of the wave problem and create a data-set of sensors recordings from many different arbitrary obstacles place inside the domain. We formulated this as a datadriven problem and proposed a method for solving the inverse problem of recovering the obstacle from the sensor measurements. The method we proposed has three blocks: a) using TR to backward propagate the

Method	Number of train samples	Mean IOU	Median IOU	IOU standard deviation
Non-TR [11]	22,500	66%	-	-
Proposed method	9,000	60.96%	67.64%	0.21
Proposed method	18,000	65.88%	72.78%	0.22
Proposed method	27,000	68.21%	75.6%	0.21
Proposed method	54,000	71.69	79.27	0.21

Table 1: Statistical aggregation of the obstacle identification results. The metric is the IOU.

solution and recover the initial condition, b) use a deep neural network to extract the obstacle from this reconstruction, and c) a physics-informed loss term that utilizes the forward wave problem to penalize bad predictions made by the network. We show that the method is able to accurately recover the location, shape and size of the obstacle and even compete with the reference methods available in the literature. In this work we did not introduce measurement noise, and we are currently investigating extending this method to work with high measurement noise in the sensors.

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