Viscous/Inviscid Unsteady Flow Simulations Using Third Order in Time Filtered BDF-2

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Abstract: To increase the order of the time-discretization without significant additional computational work, we consider adding a pre- and/or post- filtering step as suggested in [1]. We develop and implement pre- and post-filtering of the second order Backward Differentiation Formula (BDF) scheme for simulations of the Navier-Stokes/Euler equations characterized by a slow varying characteristic times.

Keywords: Navier-Stokes, Backward Differentiation Formula,General Linear Methods, High order scheme, filtering.

1 Introduction

To improve the efficiency one can use high order spatial schemes in order to reduce the mesh size and/or use an implicit scheme to allow the use of large time steps while maintaining accuracy and stability, To keep the accuracy we need a high order temporal integration scheme. Implicit RK is a good candidate for the high order scheme. However, it requires internal stages which cost CPU time. The BDF scheme is a commonly used one-step implicit scheme. However, for A-stability it can be at most second order. We add a pre- and/or post-filtering step so that the BDF is increased to third order accuracy, without significant additional computational work. Results will be presented for both inviscid and viscous flow simulations.

2 Numerical method

We consider the three dimensional, compressible Navier-Stokes equations in conservative form and solve it with a dual-time stepping method. In space we use a finite volume approach. We approximate the physical time derivative by a second order BDF and get the semi-discrete scheme

$$\frac{\partial W}{\partial \tau} = \frac{3W^{n+1} - 4W^n + W^{n-1}}{2\Delta t} - \sum_{cell faces} (\vec{F} - \vec{F}_v) \cdot d\vec{S} = -R^*(W) \tag{1}$$

W is the conservative variables, τ is the pseudo-time, R^* is the residual, F is the inviscid flux and F_v is the viscous flux. For each physical time step we solve (1) using a Runge-Kutta scheme in pseudo-time replacing the residuals $R^*(W^{(k)})$. $W^{(k)}$ is the solution in the k-th RK step, by a smoothed residual (see for example [2]).

To increase the order of the time-discretization, without significant additional computational work, we add a pre- and/or post- filtering step by including previous time-steps. In recent work [1] families of general linear methods (GLM) have been used to filter a variety of methods. They characterized the behavior of the coefficients of the equivalent GLM in terms of the filter parameter and the GLM starting point. They used this characterization to optimize properties of the equivalent GLM by choosing the filter parameters. For example, a BDF scheme can be pre- and post-filtered with the inclusion of two prior steps and written in the GLM form

$$\begin{aligned} y^{(1)} &= d_1 u^{n-3} + d_2 u^{n-2} + d_3 u^{n-1} + d_4 u^n \quad \text{(Pre-filter)} \\ y^{(2)} &= -\frac{1}{3} u^{n-1} + \frac{4}{3} y^{(1)} + \frac{2}{3} \Delta t F(y^{(2)}). \\ u^{n+1} &= \theta_1 u^{n-3} + \theta_2 u^{n-2} + \theta_3 u^{n-1} + \theta_4 u^n + b \Delta t F(y^{(2)}) \quad \text{(Post-filter)} \end{aligned} \tag{2}$$

To enlarge the $A(\alpha)$ stability region while increasing the order of accuracy we test a four step, three stage, method with a linear stability region $A(\alpha)$ with $\alpha \approx 89.62$, given in [1]. The coefficients are:

$$d_1 = 2.670130894410204, d_2 = -3.311517498805319, d_3 = -3.489799303077245,$$

$$d_4 = 5.131185907472361, \theta_1 = 0.370742163920604, \theta_2 = -0.631064728171402,$$

$$\theta_3 = -0.729528261935270, \theta_4 = 1.989850826186068, b = 0.120568773483737.$$

The values $y^{(1)}$ and $y^{(2)}$ correspond to the solution at the intermediate time-levels $t^n + c_1$ and $t_n + c_2$, respectively, $c_2 = 3.930023404911324$, $c_1 = c_2 - \frac{2}{3}$. This method was optimized for $A(\alpha)$ stability. The linear stability region includes the entire real axis, and the majority of the left half-plane, but not the imaginary axis.

3 Results

For a formal order verification of the method we consider the linear equation $\frac{du}{dt} = -u$ with the initial condition $u(0) = 1$. Figure 3.1 presents the error for the filtered BDF combined with a dual-time stepping approach. We see that 3rd order is achieved. Figure 3.2 presents the comparison between the solutions of a moving isentropic vortex problem, simulated with the filtered three time step BDF to the solution from a reference solution. The improvement of the solution as the time step decreases can be seen.

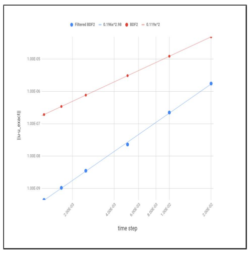

Figure 3.1: Order verification for filtered BDF for linear scalar equation.

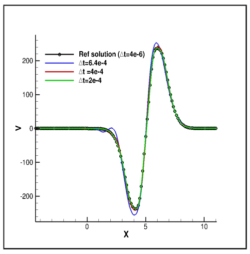

Figure 3.2: center-line cut of solution of isentropic vortex problem with filtered BDF-2

References

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