Droplet Formation Simulations Using Mixed Finite Element Method

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Abstract: Droplet formation and pinch-off dynamics is analyzed using a one-dimensional axisymmetric mixed finite element formulation, in particular for the paraffin wax used in hybrid rocket engines. The algorithm uses adaptive mesh refinement to capture singularity and runs self-consistently to calculate droplet elongation. The code is validated against laboratory experiments.

Keywords: Droplet pinch-off dynamics, Mixed finite elements.

1 Introduction

Paraffin wax is a prominent candidate among high regression rate fuels for hybrid rocket engines [1]. The atomization of the paraffin wax that begins by the droplet formation and pinch-off, enables rapid burning and generates much more specific thrust than other fuels. Understanding pinch-off dynamics for paraffin wax, and in turn predicting of droplet sizes and pinch-off times, is crucial for designing and modeling hybrid rocket engines. In this study, we create a novel finite element model for gravity-driven droplet dynamics. Our implementation incorporates self-consistent algorithm and adaptive mesh refinement. We verify our model with the Method of Manufactured Solution (MMS), and then validate it against laboratory experiments on water and glycerol droplets.

2 Problem Statement

We consider one-dimensional axisymmetric fluid column described by the Navier-Stokes equations in cylindrical coordinates as treated by Eggers and Dupont [2]. The governing equations are discretized using mixed finite elements, augmented by a new variable (s) to simplify the curvature term. The weak form using q, v, and w are test functions, is given by

$$\int_\Omega q \left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + \frac{\gamma}{\rho}\frac{\partial(\nabla\cdot\hat{\mathbf{n}})}{\partial z} - \frac{3\nu}{h^2}\frac{\partial}{\partial z}\left(h^2\frac{\partial u}{\partial z}\right) - g\right] d\Omega = 0 \tag{1}$$

$$\int_\Omega v \left[\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial z} + \frac{1}{2}h\frac{\partial u}{\partial z}\right] d\Omega = 0 \tag{2}$$

$$\int_\Omega w \left[s - \frac{\partial h}{\partial z}\right] d\Omega = 0 \tag{3}$$

Where, the curvature term is

$$\nabla\cdot\hat{\mathbf{n}} = \left[\frac{1}{h\left(1+s^2\right)^{1/2}} - \frac{\frac{\partial s}{\partial z}}{\left(1+s^2\right)^{3/2}}\right] \tag{4}$$