

Droplet Formation Simulations Using Mixed Finite Element Method

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Abstract: Droplet formation and pinch-off dynamics is analyzed using a one-dimensional axisymmetric mixed finite element formulation, in particular for the paraffin wax used in hybrid rocket engines. The algorithm uses adaptive mesh refinement to capture singularity and runs self-consistently to calculate droplet elongation. The code is validated against laboratory experiments.

Keywords: Droplet pinch-off dynamics, Mixed finite elements.

1 Introduction

Paraffin wax is a prominent candidate among high regression rate fuels for hybrid rocket engines [1]. The atomization of the paraffin wax that begins by the droplet formation and pinch-off, enables rapid burning and generates much more specific thrust than other fuels. Understanding pinch-off dynamics for paraffin wax, and in turn predicting of droplet sizes and pinch-off times, is crucial for designing and modeling hybrid rocket engines. In this study, we create a novel finite element model for gravity-driven droplet dynamics. Our implementation incorporates self-consistent algorithm and adaptive mesh refinement. We verify our model with the Method of Manufactured Solution (MMS), and then validate it against laboratory experiments on water and glycerol droplets.

2 Problem Statement

We consider one-dimensional axisymmetric fluid column described by the Navier-Stokes equations in cylindrical coordinates as treated by Eggers and Dupont [2]. The governing equations are discretized using mixed finite elements, augmented by a new variable (s) to simplify the curvature term. The weak form using q , v , and w are test functions, is given by

$$\int_{\Omega} q \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial z} + \frac{\gamma}{\rho} \frac{\partial(\nabla \cdot \hat{\mathbf{n}})}{\partial z} - \frac{3\nu}{h^2} \frac{\partial}{\partial z} \left(h^2 \frac{\partial u}{\partial z} \right) - g \right] d\Omega = 0 \quad (1)$$

$$\int_{\Omega} v \left[\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial z} + \frac{1}{2} h \frac{\partial u}{\partial z} \right] d\Omega = 0 \quad (2)$$

$$\int_{\Omega} w \left[s - \frac{\partial h}{\partial z} \right] d\Omega = 0 \quad (3)$$

Where, the curvature term is

$$\nabla \cdot \hat{\mathbf{n}} = \left[\frac{1}{h(1+s^2)^{1/2}} - \frac{\frac{\partial s}{\partial z}}{(1+s^2)^{3/2}} \right] \quad (4)$$

3 Results

The computational model is validated with previous experimental work by Zhang and Basaran [3]. Figure 1 shows the evolution of non-dimensional droplet length in time away from the pinch-off for water and 85% glycerol solution. The plot shows an excellent agreement of the numerical simulation results with the experimental data. The water droplet quickly approaches the pinch-off, causing less elongation of the neck due to very low viscosity. Compared to that, the glycerol takes more time to approach the pinch-off, resulting in longer neck caused by the damping effect of high viscosity.

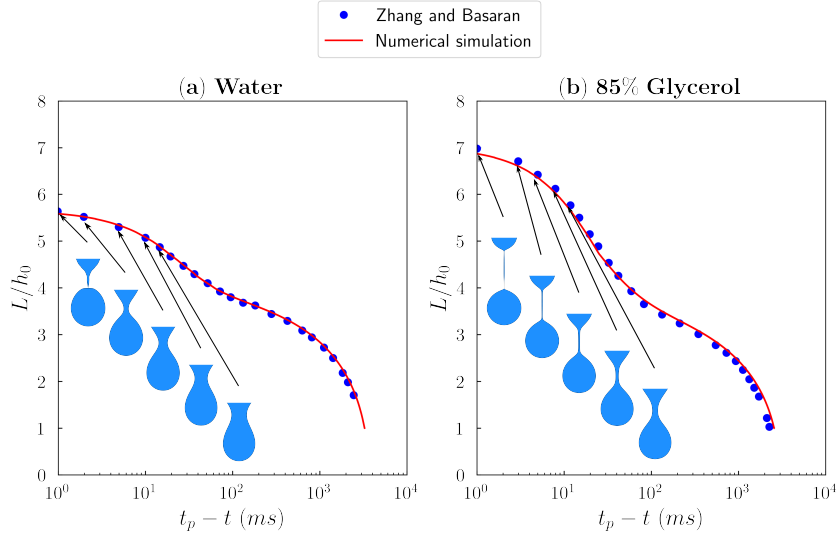


Figure 1: Evolution of non-dimensional length in time approaching the pinch-off for (a) water and (b) 85% glycerol. The inlet radius $h_0 = 0.0016$ m and inflow rate = 1 mL/min.

4 Conclusion and Future Work

A one-dimensional model is reliably accurate to simulate axisymmetric droplet formation. In future work, we expand this model to incorporate shear-driven droplet formation in a background flow. This more closely matches the process of atomization in a hybrid rocket engine, and impacts both the pinch-off time and droplet volume.

References

- [1] M. Karabeyoglu and B. J. Cantwell, “Combustion of liquefying hybrid propellants: Part 2, stability of liquid films,” *Journal of Propulsion and Power*, vol. 18, no. 3, pp. 621–630, 2002.
- [2] J. Eggers and T. F. Dupont, “Drop formation in a one-dimensional approximation of the navier–stokes equation,” *Journal of fluid mechanics*, vol. 262, pp. 205–221, 1994.
- [3] X. Zhang and O. A. Basaran, “An experimental study of dynamics of drop formation,” *Physics of fluids*, vol. 7, no. 6, pp. 1184–1203, 1995.