Multidimensional HLLC Riemann solver for the Eulerian droplet equation system

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Abstract: In clouds and under cold weather conditions, water droplets impact and freeze on aircraft structures. The Eulerian model for the air-droplet flow predicts the drolet impingement. The model equations are close to the Euler equations but without a pressure term. Consequently, the resulting system is not strictly hyperbolic and standard Riemann solvers can not be used. To circumvent this problem, the system is modified to include the divergence of a particle pressure. The main purpose of this work is to implement a multidimensionnal HLLC Riemann solver for the modified formulation of the Eulerian droplet model. The method should preserve physical properties such as the density positivity and must produce accurate results compared to existing codes.

Keywords: Multi-phase flow, Pressureless Euler equations, Riemann solver, In-flight icing.

1 Problem Statement

In-flight icing is still responsible for many crashes and accidents. Predicting the droplets impingement is one of the mandatory step needed to design aircraft ice protection devices. In clouds, the air flow around the wing transports suspended water droplets. Due to their high inertia, they impact aircraft surfaces and may freeze. The Eulerian model for the air-droplet flow involves two variable fields: the droplet velocity ($\mathbf{U}$) and the volume fraction of water (α). The mass and momentum conservation equations are

$$\begin{cases} \dfrac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{U}) = 0 \\ \dfrac{\partial \alpha \mathbf{U}}{\partial t} + \nabla \cdot (\alpha \mathbf{U} \otimes \mathbf{U}) = \mathbf{F}_a + \mathbf{F}_g \,. \end{cases} \tag{1}$$

The gravity, $\mathbf{F}_g = \alpha(1 - \dfrac{\rho_a}{\rho_w})\mathbf{g}$, and the air friction, $\mathbf{F}_a = \alpha \dfrac{3\mu_a C Re_d}{4\rho_w d^2}(\mathbf{U}_a - \mathbf{U})$, act on the droplets. Re_d is the Reynold number of the droplets in the air flow, $\mathbf{U}_a$ the air velocity, C the experimental drag coefficient, d the average droplet diameter and ρ_a, μ_a, ρ_w the air density, the air viscosity and the water density.

This Eulerian model is a weakly hyperbolic system and classical Riemann solvers are not applicable [4]. Many approaches have been developed to solve this problem such as using a

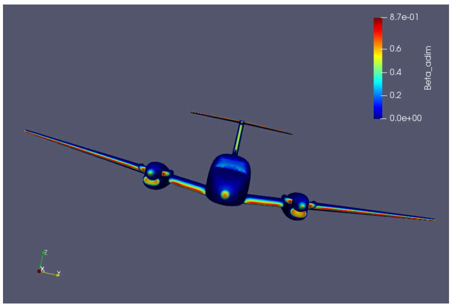

Figure 1: Local collection efficiency on a turboprop aircraft, $d = 60\,\mu m$, Mach=0.2, AoA=0^o

Jordan decomposition or introducing a small term to artificially restore hyperbolicity [1, 2]. However, if a droplet pressure is added, the system becomes strictly hyperbolic. In order to solve a mathematical problem equivalent to (1), a droplet pressure is added and subtracted to the left hand side, such that

$$\begin{cases} \dfrac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{U}) = 0 \\ \dfrac{\partial \alpha \mathbf{U}}{\partial t} + \nabla \cdot (\alpha \mathbf{U} \otimes \mathbf{U} + \alpha g d) - \nabla \cdot (\alpha g d) = \mathbf{F}_a + \mathbf{F}_g \end{cases} \tag{2}$$

The term $\nabla \cdot (\alpha g d)$ is added to the source terms. The particle pressure $\alpha g d$ in the convective terms allows the use of a classical Riemann solver.

Figure 1 shows impingement on an aircraft surface. In the shadow areas behind the aircraft, the volume fraction can be close to zero. Vacuum states can arise where $\alpha = 0$ and the velocity is undefined. Non-physical negative volume fraction may easily appears if an appropriate Riemann solver is not used. As a consequence, this issue is the main concern to develop positivity-preserving Riemann solvers. The HLLC approximate Riemann solver satisfies this property and has been chosen and adapted to this problem by [2].

2 Conclusion and Future Work

In this work, we will investigate the use of a multidimensional HLLC Riemann solver[5]. This solver should only slightly increases the computational complexity per time-step. However, it is expected to nearly double the CFL number.

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