Application of Deep Learning to Wave phenomena

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We consider the application of deep learning with a physically informed addition to several problems, using finite differences, for the wave equation. Specifically, we consider an explicit scheme that violates the CFL stability restriction on the time step for the one dimensional wave equation. Second, we construct a convolutional dispersion relation preserving (DRP) scheme for the two dimensional wave equation that improves both accuracy and stability for high wave numbers on coarse meshes. Finally, we apply deep learning to the wave equation for the inverse problem of locating either a source or an obstacle based on data at a few sensors.

First, we consider a method to solve the one-dimensional time dependent wave propagation problem in a homogeneous domain, with reflecting boundary conditions, while violating the stability condition. We numerically approximate the solution using a simple central difference Finite-Difference (FD) scheme in both space and time. The CFL condition is a necessary condition for the stability. According to the CFL condition, using a fine grid requires also a fine temporal discretization which makes the solution very expensive. Future work, for the conference, will extend this to other wave-like equations e.g. elasticity and electromagnetics and also to nonlinear systems especially the Euler equations

We propose a method, based on a physically-informed deep-learning approach, producing stable solutions, even if the Courant-Friedrichs-Lewy (CFL) stability condition is not met. We train a neural network to learn the properties of the scheme. We generate solutions synthetically from randomly chosen initial conditions using a stable scheme and sub-sample the solutions in such a way that they are unstable (the CFL condition is not satisfied). These sub-samples are the training set for the neural-network which is then tested on different synthetically generated unstable problems. Although the training part takes hours, using the non-linear scheme produced by the network takes milliseconds. We compare the solution to both an explicit and an implicit method. We also compare the method for a wave problem with a simple analytic solution, measuring the error over time to check the performance over many time steps. We improve the method by adding a physically-informed loss element to the neural network during training. We exploit the fact that the network solves the wave problem, and creates a loss term that compares the network's learned solution with another solution produced by the FD approximation. This makes the network physically aware of the problem it is trying to approximate and therefore ensures better convergence resulting in higher precision.

Second, we focus on the two-dimensional problem. We approximate the wave equation by a finite difference method. This can be described by a kernel that gives the dependence of the next time step on the neighboring points at previous time steps. We aim to find a kernel that produces accurate results even in the setting of low resolution and high frequencies. The proposed method is based on a Physically-Informed Machine-Learning (ML) approach. ML models excel in non-linear problems and usually are able to fit the problem with high accuracy and quick convergence. However, one needs to either choose a specific ML method to fit the problem, otherwise it might not converge. By combining the classical methods with advanced ideas from the field of ML we achieve better results.

The dispersion relation connects three physical attributes: the wavenumber (spatial oscillations), frequency (temporal oscillations), and propagation velocity. When numerically solving a PDE using a FD scheme, the physical dispersion relation is replaced by a numerical dispersion which can be quite different from the physical one. Numerical dispersion is dependent on the physical parameters of the problem, the choice of discretization, and the choice of FD method. Generally, numerical dispersion is more significant when dealing with high wavenumbers and negligible with low wavenumbers.

The classical FD schemes are based on a Taylor series expansion to achieve a given order of accuracy. The idea of DRP schemes is to replace classical Taylor based kernels with different ones for which the dispersion error is reduced for low resolution mesh and high wavenumbers. The method presented here is a DRP scheme. There is a variety of ways to obtain such kernels, mainly using different optimization techniques. The first DRP scheme was proposed by Tam and Webb. They used classical optimization techniques to obtain numerical schemes. Since then, many other DRP schemes have been proposed changing the method of optimization and target/loss function. These include ideas such as simulated annealing, Lagrange multipliers, and least squares, with loss functions such as the 1-norm, 2-norm, and max-norm. The properties of the various optimization methods result in a limited wavenumber bandwidth. Other methods are designed for specific wavenumbers and while performing well on these struggle with approximating the solution for other wavenumbers. Hence, one of the main challenges of DRP schemes is accurately approximating the solution in a broadband scenario. We propose a ML based framework to create a DRP scheme. We generate data composed of solutions of the two-dimensional wave equation. We then use a machine learning model to generate a single 5X5 convolutional kernel. We achieve this by training the model using neural-network inspired methodology. Furthermore, we enrich the standard machine learning method by using a target function (loss). This loss function makes the algorithm “aware” of the physical problem. Using this approach, we achieve better results.

The third problem we consider is the inverse problem of locating either a source or an obstacle. We simulate acoustic waves propagating and record the synthetic data, over time, at a small number of sensors placed in the domain. Given these measurements, the properties of the medium and the location of the sensors, we aim to find the area of the original source that generated these measurements. We create a deep-learning (DL) based solution for the inverse problems of source refocusing, which performs well even with high levels of sensor measurement noise. Inverse problems are difficult since their solution does not necessarily exist and if so, may not be unique and are sensitive to small perturbations. Since we only have information at a very few locations we have an inverse problem with very partial information. A major drawback of learning based algorithms is that, while performing well on the specific task they are designed for, they often underperform when injected with variant data. ML can be very sensitive to perturbations and noise. Hence, the presence of measurement noise in the sensors makes recreating the initial data very challenging.

We propose a deep learning based method that performs well even with high measurement noise. We incorporate the Time-Reversal (TR) method, known for being robust to noise, within the neural-network. In addition, we train a network that estimates the noise pattern of the data and classifies the large sample set into groups based on the noise pattern. Each group has a specific TR-based neural-network that trained for this specific noise pattern. We will show the performance of the method in the presence of high measurement noise.

Computational results exist and will be presented for all the cases.