Discrete exterior calculus based flow simulations on a sphere for the modeling of Solar inertial modes

R. Ayoub[1], P. Jagad[1], R. Samtaney[1], J. Philidet [2] and L. Gizon[2,3,4]
Corresponding author: rama.ayoub@kaust.edu.sa

[1] King Abdullah University of Science and Technology, Saudi Arabia.
[2] Max-Planck-Institut für Sonnensystemforschung, Justus-von-Liebig-Weg 3, 37077 Göttingen, Germany.
[3] Institut für Astrophysik, Friedrich-Hund-Platz 1, 37077 Göttingen, Germany.
[4] Center for Space Science, NYUAD Institute, New York University Abu Dhabi, Abu Dhabi, UAE.

Abstract: Low-frequency modes of oscillation in the inertial frequency range have recently been identified on the Sun. Previous studies focused on computing the linear eigenmodes. Additional work is needed to model the amplitudes and the lifetimes of the modes. To address this question, we intend to perform numerical simulations of forced 2D turbulence on a rotating sphere retaining the nonlinear term in the momentum equation. To this end, we will use a discrete exterior calculus (DEC) based method which has certain desirable conservation and mimetic properties. Such discretizations are appropriate for investigating flows dominated by long-lived coherent structures like Rossby and inertial waves, due to its unique features. We extend our DEC based numerical scheme to include a forcing term in the momentum equation.

Keywords: 2D Turbulence, Inertial waves, Discrete Exterior Calculus.

1 Introduction

A recent study [1] reports detection of solar inertial modes in some helioseismic observations. Numerical tools allowing identification and analysis of these modes have attracted interest of researchers. Gizon et al. [1] developed a 1D and 2D *linear* framework of incompressible Navier-Stokes equations and succeeded to recover a good representation compared to the observations of the equatorial Rossby modes and the high latitude inertial modes, while the critical latitude inertial modes were not well represented. To evaluate the effect of nonlinear term on the predictions, we envisage to perform a forced 2D turbulence on a rotating sphere taking into account the nonlinear term and using a DEC method. DEC [2] based numerical methods retain many of the rules/identities of its continuous counterparts and are known to exactly conserve mass, vorticity and kinetic energy. In the present work, we extend our DEC formulation of Navier-Stokes equations [3] by adding a forcing term to the momentum equation. The form of this term is given in the following section.

2 Numerical Method

For an incompressible flow of a homogeneous fluid with unit density, subjected to external forcing and large scale dissipation, the DEC notation of the governing equations, in a rotating frame of reference are as follows

$$\left[\left(-\frac{1}{\Delta t}\right) I - \frac{1}{2}\left(W_V\right)^{n+1} *_0^{-1}\left[-d_0^T\right] *_1\right]\left(U^*\right)^{n+1} + *_1^{-1} d_1^T \left(P^d\right)^{n+1/2} = F, \tag{1}$$

with

$$\begin{aligned} F = {} & \left(-\tfrac{1}{\Delta t}\right)\left(U^*\right)^n + \tfrac{1}{2}\Big[\left(W_V\right)^n *_0^{-1}\left[-d_0^T\right] *_1\Big]\left(U^*\right)^n \\ & + \tfrac{1}{2}\Big[\left(W_V\right)^{n+1} + \left(W_V\right)^n\Big] *_0^{-1} f_{dual2} + F_u^{n+1} + D^n, \end{aligned} \tag{2}$$

$$\left[d_1\right]\left(U^*\right)^{n+1} + [0]\left(P^d\right)^{n+1/2} = 0. \tag{3}$$

The forcing vector is expressed as $F_u = d_0 F_\psi$, where F_ψ is obtained from $*_0^{-1}\left[-d_0^T\right] *_1 d_0 F_\psi = F_\xi$. Here, $F_\xi(\theta, \phi, t)$ is the vector containing vorticity forcing function at the mesh primal nodes, and is represented by a Markovian process as

$$F_\xi(\theta, \phi, t^{n+1}) = R F_\xi(\theta, \phi, t^n) + \sqrt{1 - R^2}\hat{F}(\theta, \phi, t^{n+1}),$$

where R is a memory coefficient related to memory timescale and $\hat{F}$ is a vorticity source generated randomly at each time step in terms of the spherical harmonics $Y_l^m(\theta, \phi)$ as follow

$$\hat{F}(\theta, \phi, t^n) = \sum_{l=l_c-\Delta l}^{l_c+\Delta l} \sum_{\substack{m=-l \\ m\neq 0}}^{+l} \hat{F}_l^m(t^n) Y_l^m(\theta, \phi),$$

with $\hat{F}_l^m(t^n)$ an expansion coefficient of $\hat{F}$, θ the colatitude, and ϕ the longitude. Hyperviscosity is used for the large scale dissipation. The set of nonlinear equations (1) - (3) is solved using Picard's iterative method for the mass flux (U^*) and dynamic pressure $\left(P^d\right)$ degrees of freedom.

3 Conclusion and Future Work

In the present work, the numerical simulation of solar convection is motivated. We extend our DEC method and add a forcing term to the momentum equation. We will perform the simulations and compute the energy and vorticity power spectra. We will identify the inertial and Rossby waves in the flow, and compare them with the ones derived from the observations and linear models.

References

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