Physics-Based Regression vs. CFD for Hagen-Poiseuille and Womersley Flows and Uncertainty Quantification

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Abstract: CDF and its uncertainty quantification are computationally demanding. We use neural network (NN) and Gaussian Process (GP) methods to demonstrate machine learning can build efficient and accurate surrogate models with limited runs of flow models. We apply the method to Hagen-Poiseuille and Womersley flows that involve spatial and spatial-tempo responses, respectively. Training points are generated by calling the analytical solutions multiple times with evenly discretized spatial or spatial-temporal variables. Then NN and GP surrogate models are built using supervised machine learning regression. Meanwhile, we use GPU-accelerated lattice Boltzmann code to perform the same task. The results indicate that surrogate models predict the analytical solutions as accurate as CFD but much faster than CFD. We also discuss uncertainty quantification when uncertainty exists in the surrogate models and model input.

Keywords: Supervised Machine Learning, Computational Fluid Dynamics, Hagen-Poiseuille, Womersley, Surrogate Model, Uncertainty quantification.

1 Introduction

Computational Fluid Dynamics (CFD) plays an important role to solve various real-world flow systems, but a heavy computational burden often causes a trade-off with the physical accuracy. Surrogates built from machine learning regression for the expensive flow models have the potential to achieve both fast fluid simulations and high accuracy. Although surrogate models are inexpensive, they may still have some model error. When they are used for applications, the model input may also be random. It is also a need to quantify the effects of uncertainty on the model prediction. In this study, we demonstrate that physics-supervised regression can produce efficient and accurate surrogate models, which can significantly reduce the computational time without compromising the physical accuracy of CFD, if the uncertainty is properly considered.

2 Problem Statement

We develop two surrogate models for predicting steady Hagen-Poiseuille flow and unsteady Womersley flow, both of which have analytical solutions when the flow is laminar. The existence of analytical solutions allows us to demonstrate the effectiveness of surrogate models and the use of uncertainty quantification. The flow domain is a long pipe with diameter R and length L. The analytical solutions are $u = (1 - r^2)$ and $u = (1 - r^2) + (4A/\alpha^2)Real\{[1 - J_0(\alpha r i^{\frac{3}{2}})/J_0(\alpha i^{\frac{3}{2}})]e^{i\omega t}/i\}$ for Hagen-Poiseuille flow and unsteady Womersley flow, respectively, where u, r, and A are normalized velocity, distance to center, oscillating magnitude, respectively; α and J_0 are the Wormsley number and the Bessel function of the first kind of order zero, respectively. The surrogate models are built via machine learning regression. For unsteady flows, time is also included as a dimension of the input training points. The CFD is performed using volumetric lattice Boltzmann method [1] through an in-house GPU accelerated code [2]. We impose pressure gradient as a body force, rigid wall, and periodic condition at inlet and outlet. The flow is incompressible with density = 1025 kg/ m^3 and kinematic viscosity = 3.415×10^{-6} m^2/s. We compare the surrogate prediction and CFD simulation for the velocity profile along a radius, with the analytical solutions. The results are plotted in Fig. 1 for steady

Hagen-Poiseuille flow and Fig. 2 for unsteady Womersley flow. For the steady flow, both surrogate prediction and CFD simulation achieve nearly identical profiles to the analytical solutions. For the unsteady Womersley flow, all the velocity profiles from surrogate model at representative time points in an oscillation are again identical to the analytical solutions whereas CFD results have noticeable deviations from the analytical solutions. The prediction error can be estimated by the model uncertainty represented by the standard deviation of the prediction if the surrogate model is created by Gaussian Process regression. It is also possible to estimate prediction error for a surrogate model built with neural network regression [3].

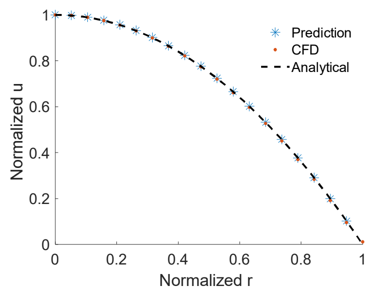

Figure 1: Steady Hagen-Poiseuille flow

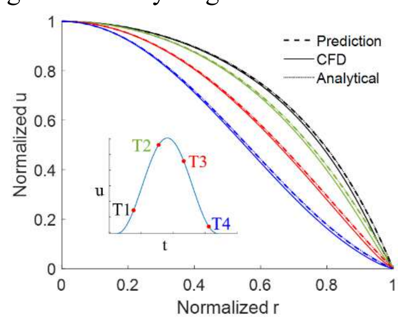

Figure 2: Unsteady Womersley flow

3 Conclusions and Future Work

In all the cases we tested, the computational time of surrogates is much less than that of CFD. The predictions from surrogates are also accurate. Uncertainty quantification results can also help users understand the effects of uncertainty on the prediction, assisting more reliable decision making. Our further work will be developing general surrogate models based on CFD samples, aiming to significantly reduce the computational time. We will train surrogate models for more realistic steady and unsteady flows and perform a full-scale uncertainty quantification analysis.

References

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Acknowledgment

The research is supported by NSF through grant CBET 1803845. This work used the Extreme Science and Engineering Discovery Environment (XSEDE), which is supported by National Science Foundation Grant No. ACI-1548562. The first author would like to also acknowledge the IUPUI University Fellowship.