

# Flow field reconstruction for Inhomogeneous Turbulence using data and physics driven models

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**Abstract:** A new methodology to construct three-dimensional, temporally stationary but spatially inhomogeneous, incompressible turbulence is presented. The method combines use of the data-driven spectral proper orthogonal decomposition (SPOD) to identify and isolate large-scale coherent motions of the flow, together with a physics-based enrichment algorithm using spatiotemporally localized Gabor modes that capture the inertial subrange turbulence. This fusion of data-driven and physics-based methods enables a statistically correct reconstruction of broadband turbulent flows using fewer modes than would be required using SPOD alone. To demonstrate the approach, we consider the problem of reconstructing wake turbulence on a plane downstream of a dragging actuator disk impinged by homogeneous isotropic turbulence. The reconstructed flow has single- and two-point correlations that are consistent with the reference high-resolution simulation data and could be used to generate statistically consistent inflow boundary conditions for subsequent simulations.

*Keywords:* Turbulence, Data driven modeling

## 1 Introduction

In this paper, we propose to use Gabor modes to enrich the low-order SPOD flow reconstruction in order to recover truncated portions of the flow. Gabor modes are compact support wavepackets, with each mode carrying its own real valued wavevector,  $\mathbf{k}$ , a real valued spatial location,  $\mathbf{x}$ , and complex valued velocity vector,  $\hat{\mathbf{u}}$ . As shown by Ghate & Lele (2020)[1], representation of subfilter-scale turbulence using Gabor modes has three distinct advantages: (a) the modes are equivalent to a POD basis for quasi-homogeneous or locally homogeneous turbulence, and such a discrete modal representation can be obtained using a very small number of unique modes, (b) temporal evolution of the modes can be derived as a WKB-asymptotic approximation to subfilter-scale Navier–Stokes equations, and is equivalent to solving an ordinary differential equation (ODE) for each mode travelling in a Lagrangian frame of motion, and (c) the transform from modal/wavespace to physical space can be performed efficiently using non-uniform fast Fourier transforms (NUFFT). Critically, the Gabor modes used to enrich the SPOD reconstruction are a function only of the retained SPOD modes, and therefore do not



Figure 1: Instantaneous snapshot of longitudinal velocity fluctuations for a turbulence wake shown on a fixed streamwise cross-sectional plane. The figure

increase the order of the reconstruction. In summary, the method proposed herein enables flow field reconstruction using fewer SPOD modes.

## 2 Problem Statement

The problem being considered is that of interaction between broadband isotropic turbulence and a dragging actuator disk. This set-up was originally studied using SPOD by Ghate et al. (2018)[2]. The algorithm proposed is illustrated in Figure 1. The focus is to reconstruct the velocity field at a location  $\mathbf{x}$  at a time,  $t$  as:

$$\mathbf{u}(\mathbf{x}; t) = \mathbf{u}^{\text{Mean}}(\mathbf{x}) + \mathbf{u}^{\text{SPOD}}(\mathbf{x}; t) + \mathbf{u}^{\text{Gab}}(\mathbf{x}; t) \quad (1)$$

where the time averaged field,  $\mathbf{u}^{\text{Mean}}$  is assumed to be known (can be generated via a separate Reynolds averaged Navier-Stokes simulation). The field variable,  $\mathbf{u}^{\text{SPOD}}$  is a stochastic field generated using precomputed SPOD modes that are trained using the LES dataset in [2]. Finally, the field  $\mathbf{u}^{\text{Gab}}$  is also a stochastic field but generated via physics based Gabor modes. Accuracy of the reconstruction is evaluated via comparisons of second order two-point space-time correlations with the baseline LES dataset used for training the SPOD modes.

## 3 Conclusion and Future Work

A flow reconstruction method that combines data-driven modal analysis with physics-based turbulence enrichment is developed and tested for incompressible wake turbulence. Among broader use cases of the proposed method include generation of ensembles of statistically equivalent inflow conditions containing both inhomogeneous large-scale and homogeneous small-scale motions.

## References

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