An Adaptive Space-Time Hyperbolic Navier-Stokes Solver for Two-Dimensional Unsteady Viscous Flows

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Abstract: In this paper, we present an adaptive-grid space-time solver based on a hyperbolic Navier-Stokes formulation for two-dimensional unsteady viscous flows, where the two-dimensional Navier-Stokes equations are discretized and solved as a steady system in a three-dimensional space-time domain with the coordinates (x, y, t), where t denotes time, using adaptive tetrahedral grids. The hyperbolic viscous formulation drastically simplifies the viscous discretization and greatly improves the accuracy and quality of derivatives on adaptive grids with convergence acceleration. These advantages are demonstrated for space-time computations for two-dimensional unsteady viscous flows.

Keywords: Hyperbolic Navier-Stokes, Space-Time, Adaptive Grids.

1 Introduction

Space-time methods, where unsteady governing equations are solved in a larger space with the physical time treated as an additional coordinate, are promising approaches for exploiting an ever-increasing parallelism. The methods are especially efficient when combined with anisotropic grid adaptation as we demonstrated in our previous papers [1,2]. However, solution derivatives obtained on adaptive grids are contaminated with numerical noise, which is a serious issue for viscous simulations, where target quantities largely depend on derivatives (e.g., viscous stresses and heat fluxes). To address this issue, we develop a new space-time solver based on a hyperbolic Navier-Stokes formulation [3], which has been known to help design a discretization giving superior accuracy and quality in the solution derivatives on irregular tetrahedral grids. This paper shows its application to adaptive-grid space-time computations. In this paper, we consider the hyperbolic Navier-Stokes formulation called HNS20G [3] in two dimensions:

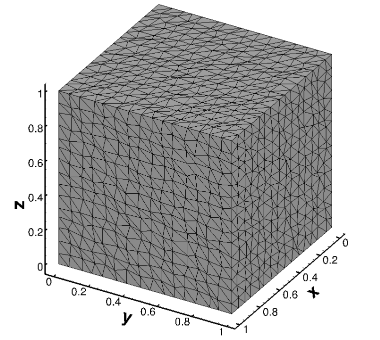
$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}_x}{\partial x} + \frac{\partial \mathbf{f}_y}{\partial y} = 0, \tag{1}$$

where $\mathbf{u}$ is a vector of conservative variables and the gradients of the primitive variables (12 variables in total), $\mathbf{f}_x$ and $\mathbf{f}_y$ are fluxes in the x- and y-directions. This system is discretized entirely by an upwind method and solved as a steady system in a three-dimensional space with the coordinates $(x, y, z = t)$.

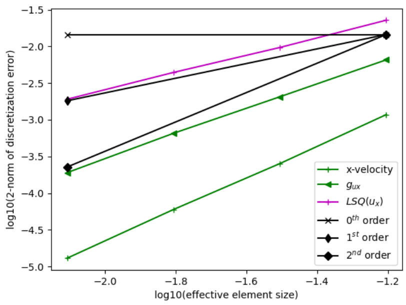
2 Preliminary Results

We developed and implemented a hyperbolic Navier-Stokes space-time solver based on the node-centered edge-based method in NASA's FUN3D code. Preliminary results are available for accuracy verification. Using a series of consistently-refined irregular tetrahedral grids (see Figure

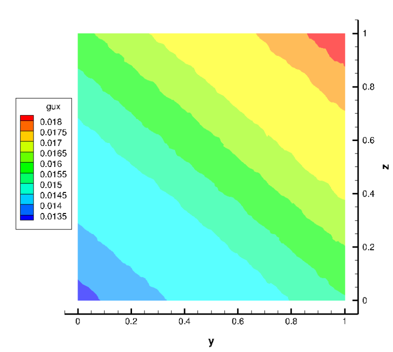
1(a) for the coarsest grid) in a unit cube, we solved the system of residual equations and verified the order of discretization error convergence. Error convergence results are shown in Figure 1(b) for the x-velocity u, g_{ux}, which is an extra variable corresponding to its x-derivative (results are similar to other variables), and the derivative computed by a least-squares (LSQ) method applied to u denoted by $LSQ(u_x)$. As can be seen, both u and g_{ux} are second-order accurate while $LSQ(u_x)$ is only first-order accurate and has much greater error. Also as can be shown in Figures 1(c) and 1(d), contours of g_{ux} is much smoother than those of the LSQ gradient, $LSQ(u_x)$ (obtained on the third grid with 64^3 nodes). These results are encouraging and demonstrate the potential of the HNS20G solver for accurate and efficient viscous space-time simulations with adaptive grids.



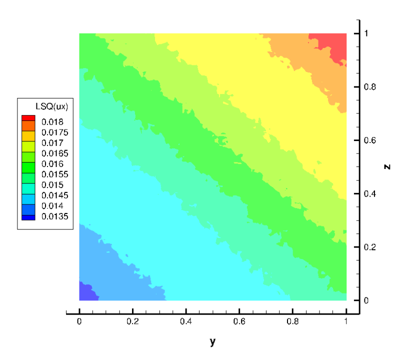
(a) Coarsest grid.



(b) Error convergence.



(c) $g_{ux}(= \partial_x u)$ at $x = 0.5$.



(d) $LSQ(u_x)$ at $x = 0.5$.

Figure 1: Accuracy verification results.

3 Final Paper

In the final paper, we plan to add results for an unsteady flow over a circular cylinder with detailed comparisons with a conventional space-time viscous solver, in terms of gradient accuracy and computational cost.

References

[1] E. Padway and H. Nishikawa, AIAA Paper 2022-2201, AIAA SciTech 2022 Forum, 3-7 January 2022, San Diego & Online.

[2] E. Padway and H. Nishikawa, AIAA Paper 2020-3024, AIAA Aviation Forum, 15-19 June 2020, Virtual Event.

[3] L. Li, J. Lou. H. Nishikawa and H. Luo, "Reconstructed discontinuous Galerkin methods for compressible flows based on a new hyperbolic Navier-Stokes system", Journal of Computational Physics, Volume 427, 15 February 2021, 110058.