The Harmonic Linearized Navier-Stokes Equations for Transition Prediction in Three-Dimensional Flows

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Abstract: The conventional method to predict the onset of laminar-turbulent transition in convectively unstable boundary-layer flows is based on the logarithmic amplification ratio, the so-called N-factor, of the linear instability waves. To calculate the N-factor, the flow variables are decomposed into a laminar basic state solution and the linear disturbances, which are assumed to be harmonic in time. The most commonly used linear stability analysis approaches include the locally parallel linear stability theory (LST) and the nonlocal, weakly nonparallel parabolized stability equations (PSE). However, these methods do not account for strong streamwise gradients that are encountered in several configurations of interest, as roughness elements, steps, gaps, or corners. To solve the linear evolution of disturbances along such strongly nonparallel regions, the harmonic linearized Navier-Stokes equations (HLNSE) need to be solved. The discretization of the HLNSE for spanwise/azimuthally inhomogeneous laminar basic states yields a linear system of complex arithmetic with a leading dimension of the order of 10^7 to 10^8. A combined multithread and multiprocessor algorithm is implemented for the direct solution of such linear system. Results for a supersonic boundary layer over a three-dimensional roughness patch show good agreement with experimental measurements when the evolution of the instability waves over the roughness patch is included via the HLNSE.

Keywords: Boundary Layer Stability, Numerical Algorithms.

1 Problem Statement

Linear stability analysis theory is based on the decomposition of the flow variables, $\mathbf{q}(x, y, z, t)$, into a laminar basic state, $\bar{\mathbf{q}}(x, y, z)$, and the unsteady perturbations, $\tilde{\mathbf{q}}(x, y, z, t)$. The linear perturbations are assumed to be harmonic in time and are written as $\tilde{\mathbf{q}}(x, y, z, t) = \check{\mathbf{q}}(x, y, z) \exp(-\mathrm{i}\omega t)+$ c.c., where ω is the angular frequency, and c.c. refers to the complex conjugate [3]. The disturbance functions $\check{\mathbf{q}}(x, y, z)$ satisfy the HLNSE, $\mathbf{L}\check{\mathbf{q}}(x, y, z) = \check{\mathbf{f}}$, where the linear operator $\mathbf{L}$ depends on the basic state variables and global flow parameters, as well as the disturbance frequency ω, and $\check{\mathbf{f}}$ represents an external forcing in the form of boundary and/or volumetric forcing.

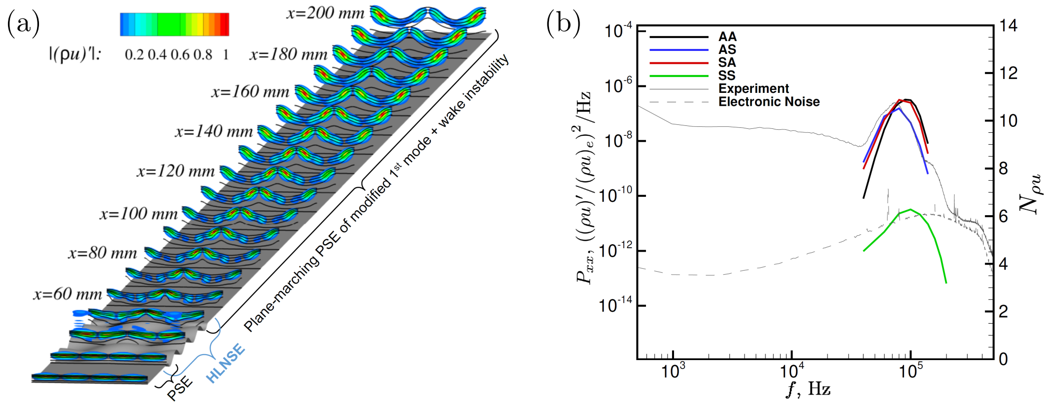

Figure 1: (a) Streamwise mass flux contours of 80 kHz disturbance and basic state isolines over the rough plate at Mach 3.5. (b) Comparison of power spectra to most-amplified disturbances at $x = 153.8$ mm distance from the leading edge. The disturbance evolution is calculated with a combination of PSE and HLNSE. Reproduced from Ref. [2]

The structure of the matrix operator **L** depends on the form of the discrete spatial derivative operators. In the present study, we use a fourth-order, centered finite difference (FD) scheme to discretize the HLNSE along the streamwise direction, while high-order FD schemes are used along the wall-normal and spanwise directions. Because of the selected streamwise discretization, the matrix **L** becomes a block pentadiagonal matrix [1]. The direct solution of the linear system of equations is obtained by using a combined multithread and multiprocessor approach based on the Thomas algorithm and the dual Schur complement method. This algorithm reduces the scaling of the cpu time and memory requirements with respect to the number of streamwise points from cubic to linear and from quadratic to linear, respectively.

A combination of the PSE and HLNSE is used to study the evolution of linear disturbances over a flat plate with a sinusoidal roughness path at the conditions of the Mach 3.5 quiet tunnel to support an experimental campaign [2]. The evolution of the 80 kHz AA disturbance, i.e., with antisymmetric boundary conditions in both symmetry planes, is shown in Fig. 1(a). The favorable agreement with the measured power spectra in Fig. 1(b) confirms the significance of disturbance growth over the roughness patch and the effect on the instabilities in the wake.

The final paper will include the details of the numerical algorithm used to solve the HLNSE, as well as additional applications.

This material is based upon research supported by the Hypersonic Technology Project (HTP) and the NASA Transformational Tools and Technologies (TTT) project of the Transformative Aeronautics Concepts Program under the Aeronautics Research Mission Directorate.

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