A Moving Discontinuous Galerkin Finite Element Method with Interface Condition Enforcement for Compressible Multi-material Flows

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Abstract: A moving discontinuous Galerkin (DG) finite element method with Interface Conservation Enforcement (MDG-ICE) is further developed and extended for solving compressible multi-material flow problems, where both conservative quantities and grid geometry are considered as independent variables. A space-time DG formulation is used to solve the multi-material compressible Euler equations in the standard discontinuous solution space and the discrete grid geometry is solved using a variational formulation in a continuous space. A self-adaptive Levenberg-Marquardt method is utilized to solve the resulting over-determined system of nonlinear equations arising from the MDG-ICE formulation. A number of numerical experiments are conducted to assess the accuracy and performance of the MDG-ICE method. Numerical results obtained indicate that the MDG-ICE method is able to deliver the designed order of both *h*- and *p*-convergence even for discontinuous solutions, and detect all types of interfaces, via interface condition enforcement and satisfy, via grid movement, the compressible multi-material Euler equations and the associated interface condition.

Keywords: Moving Discontinuous Galerkin Methods, Interface Conservation Enforcement, Multi-material Compressible Flows.

1 Introduction

The discontinuous Galerkin (DG) finite element methods are widely used in computational fluid dynamics. The discontinuous Galerkin methods have many attractive advantages like 1) its ability to achieve high-order (>2nd) accuracy on fully unstructured grids; 2) useful mathematical properties with respect to conservation, stability and convergence; 3) its adjoint consistency to be powerful for adjoint-based optimization. In addition, the methods can also handle non-conforming elements, where the grids are allowed to have hanging nodes. Furthermore, space-time discontinuous Galerkin methods provide discretization of systems of conservation laws by simultaneously discretizing space and time. Like other DG methods, the space-time DG method also offers the prospect of both arbitrary-order accuracy in space and time and adjoint consistency. However, the DG methods have a number of weaknesses that have not yet be addressed. Besides of computational cost and storage requirement, one aspect is how the properties behave in flows that are not smooth and contain discontinuous interfaces, such as material interface and shocks. Even though DG explores a set of discrete function space with discontinuous, piecewise polynomials and it can represent the discontinuous interfaces in principal, this requires that the interfaces be aligned with the grids. The stability of the DG approach may fail when misaligned grid is used. Actually, it has been an issue in how to effectively control spurious oscillations in the presence of strong discontinuities. Recently, a moving discontinuous Galerkin finite element method with interface condition enforcement, termed MDG-ICE, was developed by Corrigan et al. for compressible flows with interfaces [1]. Unlike the traditional DG methods, this MDG-ICE method treats both conservative quantities and discrete grid geometry as independent variables. A space-time DG formulation is used to solve the governing equations in the standard discontinuous solution space, and the geometry variables are determined by enforcing the interface condition in its discontinuous solution trace space. Two attractive features of the MDG method, among others, are 1) no strategies in the form of a limiter or an artificial viscosity are required to eliminate spurious oscillations in the vicinity of discontinuities and thus maintain the nonlinear stability of the DG methods, as interfaces are detected by the interface condition enforcement, and tracked by the grid movement and the interface condition; and 2) no numerical fluxes in the form of a Riemann solver are needed to maintain linear stability of the DG methods. However, this MDG-ICE formulation can lead to an over- or under-leads t-determined system of nonlinear equations.

A variant of the MDG-ICE formulation [2], was developed to solve the conservation laws, where space and time are not treated in the same way. Meshes are set uniform in time, i.e., cannot move in the t-direction. Furthermore, the geometric variables are determined by enforcing the interface conservation using a continuous variational formulation. Our numerical experiments demonstrate that this MDG-ICE method can achieve an exponential rate of convergence for Sod and Lax-Harden shock tube problems and obtain highly accurate solutions without overheating to both double-rarefaction wave and Noh problems. The objective of the efforts presented in this work is to extend and further develop this MDG-ICE method for compressible multi-material flows. Numerical experiments for a number of benchmark test cases indicate that our MDG-ICE method is able to deliver the designed order of accuracy for multi-material flow problems, detect interfaces, via interface condition enforcement and satisfy, via grid movement, the conservation law and its associated interface condition. As an illustrative example, numerical results for a two-material supersonic flow past a wedge, which forms the problem of so-called regular refraction with a reflected shock wave, are presented in Figure 1 where one can observe that our MDG-ICE method is practically able to obtain the analytical solution, fitting both the transmitted shock and weak reflected shock, and forming an RRR shock refraction patters on slow/fast interfaces exactly. The description of the developed MDG-ICE method and numerical results for a number of test cases will be presented in the final manuscript.

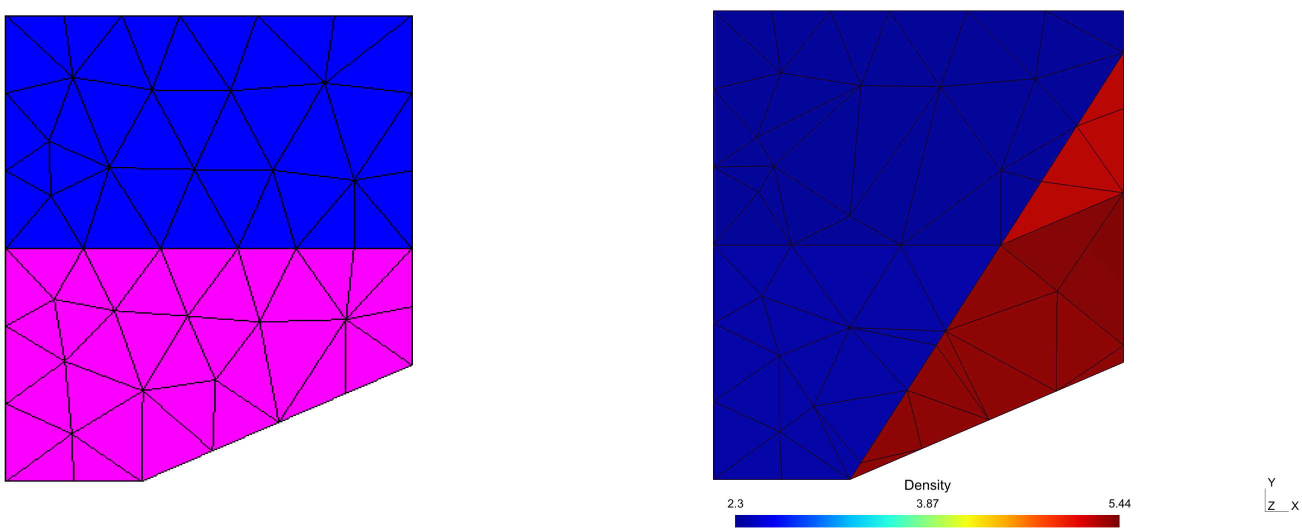

Figure 1: Initial grid and density field (left) and final converged mesh and density field obtained by the MDG-ICE solution (right) Shock wave refraction problem at a slow-fast gas interface

References

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