Discontinuous Galerkin Methods for Multi-Material Shock Hydrodynamics[1]

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Abstract: This work establishes a DG method for three-dimensional multi-material flows, on unstructured tetrahedral meshes. A novel discontinuous Galerkin (DG) method with Hyperbolic-Tangent Interface Capturing (THINC) for the single-velocity multi-material (*two or more* materials) system is presented. The physical system considered here assumes stiff velocity relaxation, but no pressure and temperature equilibrium between the multiple materials. Second- and third-order DG methods are presented. A well-balanced DG discretization of the non-conservative system is proposed, and is verified by numerical test problems. To ensure strict conservation of material masses and total energy at the discrete level, a consistent interface strategy is implemented. Comparisons with the second-order finite volume method show that the DG method results in more accurate solutions for multi-material problems. With the help of numerical experiments, it is demonstrated that the DG method shows great potential in the field of multi-material hydrodynamics.

Keywords: Discontinuous Galerkin, Non-Equilibrium Multi-Material.

We present a high-order diffuse interface method for multi-material hydrodynamics. Diffuse interface methods cause interfaces to smear over multiple computational cells. A large effort has been directed towards interface sharpening via algebraic methods using compressive limiting functions. We leverage one such approach in this work, specifically the THINC method [1]. We combine the THINC reconstruction with the multi-material Discontinuous Galerkin (DG) discretization. This results in a method that benefits from the strengths of THINC and DG: 1) the DG high-order solution can compute highly accurate solutions in smooth regions of the domain; 2) the interface capturing capabilities of THINC can resolve interfaces sharply *and* efficiently. A novel DG method for one-dimensional non-equilibrium multi-material hydrodynamics was recently proposed [2]. In this work, we extend this method to three-dimensional systems on unstructured meshes, and demonstrate the benefits for multi-material hydrodynamics.

The velocity equilibrium limit of the multi-material hydrodynamic equations [3] can be ex-

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pressed as, $\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}_j}{\partial x_j} + \boldsymbol{D} = \boldsymbol{S}$, where,

$$\boldsymbol{U} = \begin{bmatrix} \alpha_k \\ \alpha_k \rho_k \\ \overline{\rho} u_i \\ \alpha_k \rho_k E_k \end{bmatrix}, \quad \boldsymbol{F}_j = \begin{bmatrix} 0 \\ \alpha_k \rho_k u_j \\ \overline{\rho} u_i u_j + \overline{p} \delta_{ij} \\ \alpha_k \rho_k E_k u_j \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} u_j \frac{\partial \alpha_k}{\partial x_j} \\ 0 \\ 0 \\ \alpha_k p_k \frac{\partial u_j}{\partial x_j} + Y_k u_j \frac{\partial \overline{p}}{\partial x_j} \end{bmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} S_{\alpha,k} \\ 0 \\ 0 \\ -\overline{p} S_{\alpha,k} \end{bmatrix}.$$

where $k = 1, 2, ..., m$ where m is the number of materials. α_k is the volume-fraction of material-k. Bulk properties are represented $\overline{\rho}$ and pressure $\overline{p}$. $Y_k = \alpha_k \rho_k / \overline{\rho}$ is the mass fraction of material-k. The source term $S_{\alpha,k}$ corresponds to the volume-fraction redistribution due to compaction, owing to the different compressibilities of the materials.

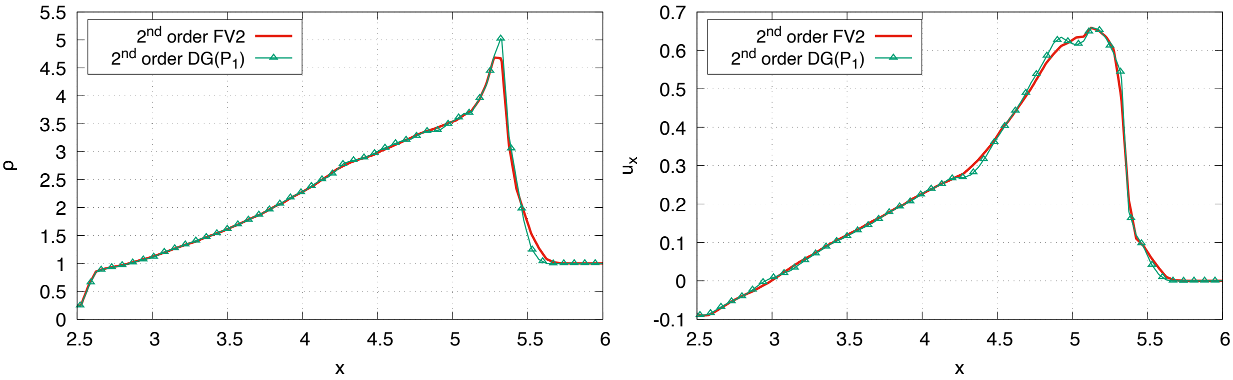

Figure 1: Triple-point problem (3 materials): Density (left) and x-velocity (right) obtained by FV2 and DG(P1) along the domain symmetry-axis

Maintaining high-order accuracy in regions of smooth flow, and monotonicity in regions of discontinuities, poses a unique challenge for multi-material hydrodynamics; because of the unit-sum constraint on the volume-fractions and their boundedness between 0 and 1, and the non-conservative nature of the equations. Furthermore, consistency between slopes of conserved quantities and volume-fractions is necessary to ensure discrete conservation of mass and energy. A limiter strategy that discretely satisfies conservation is used for this. The resulting method shows high solution accuracy (see Fig. 1), especially in smooth regions. This abstract provides a brief overview of the proposed multi-material method. Further details about the numerical method and more complex test problems will be presented in the final paper.

References

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