High-Order Cut-Cell Methods for High-Fidelity Flow Simulations

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Abstract

Cut-cell methods allow for the use of Cartesian meshes to resolve phenomena occurring in complex geometries. The advantage of cut-cell methods over approaches requiring body fitted meshes (either structured or unstructured) is that the former do not require a costly mesh generation step or complicated code infrastructure. However, cut-cell methods have been typically limited to low orders of accuracy. In the present work, we extend our previous efforts to expand our unique high-order, stable and conservative cut-cell method to three-dimensional geometries. To test their efficacy for flow problems, the schemes are used for elliptic, parabolic, and hyperbolic systems with simple boundary conditions.

Introduction

The cut-cell method [3] allows for the solution of partial differential equations (PDEs) defined on complicated domains to be computed numerically on simple Cartesian meshes. This method has seen extensive use in the fluids community, so we define the domain of interest, Ω_f, as the fluid domain which is bounded by $\Gamma_f \cup \Gamma_s$, where the Cartesian and solid object boundaries are given by Γ_f and Γ_s, respectively. A schematic of this is shown in Fig. 1. Thus, the non-Cartesian physical boundaries are embedded into the simpler Cartesian mesh leading to computational cells which have been cut by the embedded object. Rather than modifying the physical equations to implicitly account for this object, the cut-cell approach modifies the discrete derivative operators and imposes boundary conditions directly on Γ_s.

The allure of cut-cell type methods has attracted the attention and effort of a number of researchers for many years (see [4] for a review). In theory, cut-cell methods obviate the need for unstructured meshes and allow for the use of robust, accurate and conservative finite difference/volume schemes with only slight modifications near the boundary. However, the current solutions to the severe numerical challenges of cut-cell schemes typically lead to significant modifications of both the discrete algorithms and the physical equations. The discrete algorithms are modified by requiring significant extra procedures to evaluate derivatives near the boundary since a straightforward evaluation leads to instabilities. The physical equations are typically modified by requiring some sort of stabilization procedure which manifests itself as a source term in the governing equations (even if not explicitly written as such).

In recent work [1, 2], we have demonstrated an approach to cut-cell methods that uses an offline optimization procedure to achieve stability, rather than relying on ad-hoc stabilization procedures. This has allowed for the development of discretizations of up to 8^{th} order.

Preliminary Results

Extensive stability analysis of the cut-cell schemes for planar geometries is shown in Refs [1, 2]. Initial work on extending the method to three dimensional geometries can be seen in Fig. 1. To

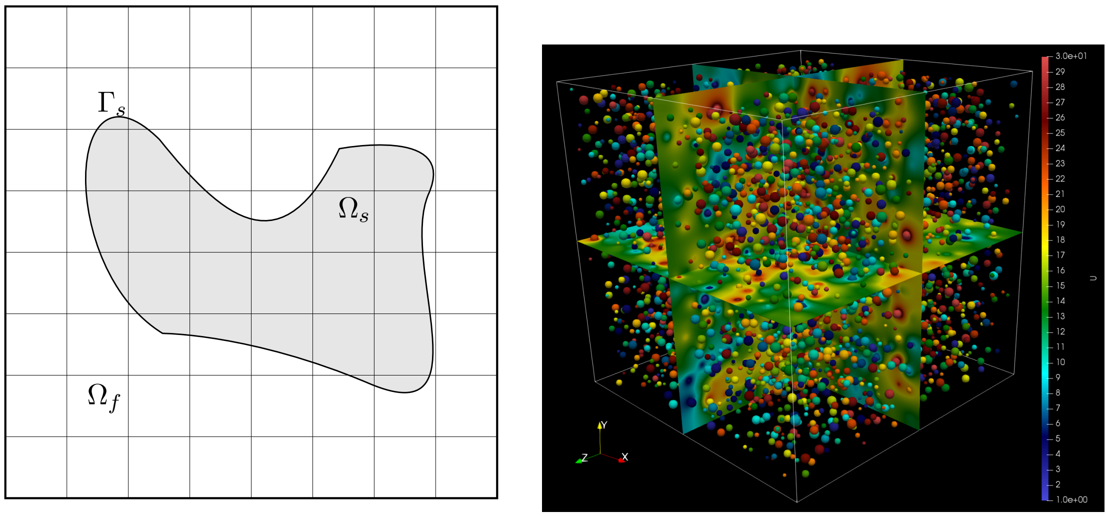

Figure 1: Left: Schematic of solid object, bounded by Γ_s, embedded in a fluid domain, Ω_f. Right: Solution to the Laplace equation, resolving 5,000 randomly placed spheres using the 8^{th} order cut-cell method. The spheres are randomly assigned a Dirichlet boundary condition in the range $[1, 30]$.

demonstrate the efficacy of the proposed approach, the Laplace equation is solved on a computational domain of 1024^3 . Neumann boundary conditions are imposed on the domain walls. 5,000 spheres are placed at random locations into the domain and resolved using our 8^{th} order cut-cell approach. The spheres are assigned randomly with chosen Dirichlet boundary conditions in the range $[1, 30]$. The slices through the domain show that the computed solution is smooth even though no small-cell correction is used. We will present 3D results covering a range of PDEs and boundary conditions along with rigorous convergence studies.

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