A Polydisperse Gaussian-Moment Method for Extended Statistical Modelling of Multiphase Flows

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Abstract: A novel model is proposed for the Eulerian treatment of particle-laden multiphase flows characterized by a dilute particle phase with an arbitrary number of distinguishable properties, such as size or temperature, based on an entropy-maximization argument. Unlike previous formulations, this new moment-based model provides a set of first-order robustly-hyperbolic balance laws that include a direct treatment for the local statistical variance of each variable, as well as the covariance between the internal variables or the internal variables and particle velocity. Two variations of the model are implemented in a massively parallel three-dimensional discontinuous-Galerkin-Hancock framework and the predictive capabilities of the new model are investigated for a range of problems, including multiphase flows resembling the dispersion of aerosol droplets generated by sprays or violent respiratory events, e.g., coughs.

1 Introduction and Scope of Work

The accurate prediction of multiphase flow when particles are differentiated by a set of distinguishable properties, such as size or temperature, can pose modelling and numerical challenges. The focus of this work is to explore two different formulations of a recently proposed polydisperse Gaussian-moment model (PGM). These models, building upon the recent PGM formulation of Forgues *et al.* [1], are described by fifteen first-order hyperbolic balance laws that provide an Eulerian treatment for higher-order statistics describing a particle phase in a background flow. The PGM formulation is an extension of the well-known ten-moment maximum-entropy model from rarefied gas dynamics and provides a direct treatment for local variances and covariances between all particle properties and velocities. This statistical information is not commonly available or utilized in traditional Eulerian methods and holds the promise of improved modelling accuracy at reduced computational cost.

Two variants of the PGM are investigated: one that assumes a log-normal distribution of particle size and one in which particle surface area is assumed to follow a normal distribution. In both cases, particle drag, buoyancy, gravitational acceleration, and evaporation are considered. There are many processes in which a log-normal distribution of the particle diameters is encountered such as the human cough [2], however, the original log-normal version of the model does not correctly recover the settling rates for particles in a quiescent atmosphere. In contrast, the surface-area-based version of the model is designed to recover this settling rate exactly.

This presentation provides the derivation of both above-mentioned forms of the PGM. The mathematical properties, including eigensystem analysis, is explored. A large-scale, three-dimensional, high-order discontinuous-Galerkin-Hancock code that efficiently solves the PGM

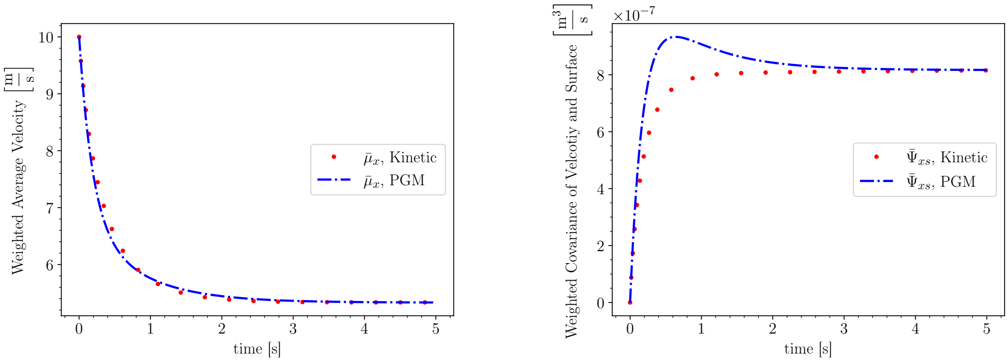
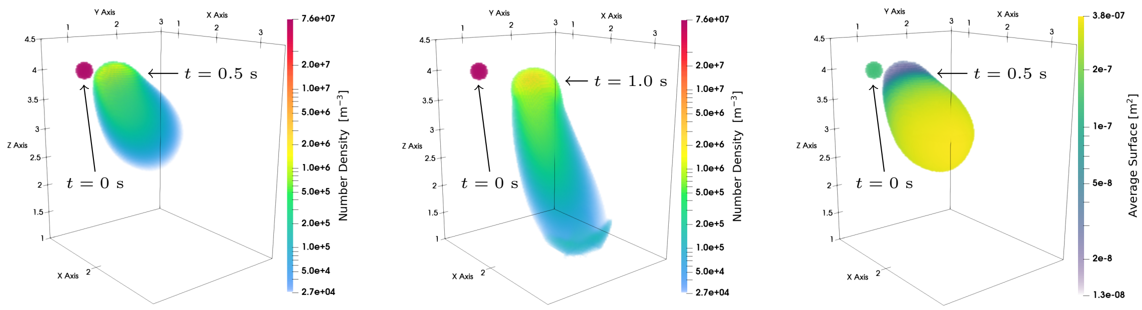


Figure 1: Space homogeneous decelerating particles



(a) Number density; t=0, t=0.5 s (b) Number density; t=0, t=1.0 s (c) Average surface; t=0, t=0.5 s

Figure 2: PGM prediction of a polydisperse puff of water droplets in an ambient environment.

using hundreds of thousands of compute cores is presented and used to compute illustrative solutions that demonstrate the capabilities of the new methods. Accuracy is demonstrated through comparisons with high-resolution Lagrangian solutions of the underlying kinetic equation describing the particle evolution.

2 Sample Results

To demonstrate the predictive capabilities of the proposed approach, two preliminary cases are included here. Figure 1 shows the time evolution of the surface-weighted average particle velocity predicted by the PGM and by a highly accurate kinetic solver (left), and the evolution of the surface-weighted covariance between velocity and particle surface area (right). Though the covariance is slightly over-predicted in the transient, the steady state is recovered exactly by the model for all moments.

Figure 2 shows the dispersion of a spherical puff of polydisperse water droplets released with a range of initial velocities, having an initial mean velocity in the positive x direction, into an ambient environment. Due to interaction with a transverse air current in the positive y direction, the puff moves diagonally, with larger particles falling more quickly due to gravity and drag. Figure 2c provides information about the location of smaller and larger particles as depicted by lower and higher values of average surface, respectively. This is enabled by the direct treatment of the local covariance between particle size and velocity.

References

[1] F. Forgues, L. Ivan, A. Trottier, and J.G. McDonald. *Journal of Computational Physics*, 398, 2019.

[2] J. Lee, D. Yoo, S. Ryu, S. Ham, K. Lee, M. Yeo, K. Min, and C. Yoon. *Aerosol and Air Quality Research*, 19(4):840–853, 2019.